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A NOTE ON DERIVATIONS OF LIE ALGEBRAS II

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Let L be a Lie algebra over a ground field of characteristic 0 and let $D(L)$ and $I(L)$ denote the Lie algebra of derivations and inner derivations of L respectively.

The following theorem is proved, although not explicitly stated, in an earlier note.[1]

THEOREM 1. *Let* $L=S+R$ *(Levi decomposition) and let* $\mathfrak{A}(S) = \{D \mid D\in D(L),\ D(S)=(0)\}$. *Further let* ρ *denote the restriction homomorphism of* $D(L)$ *into* $D(R)$. *Then* $D(L)$ *splits over* $I(L) \rightleftarrows \rho(\mathfrak{A}(S))$ *splits over* $\rho(\mathfrak{A}(S))\cap I(R)$.

If S is a semi-simple Lie algebra and M is any S-module then M is the direct sum of M^S and $S\cdot M$ where M^S is the trivial submodule of M.

THEOREM 2. *Let* $L=S+R$ *(Levi decomposition). If* $R^S\subseteq Z(R)$ *(the center of* R*) then* $D(L)$ *splits over* $I(L)$.[2]

PROOF. Let $u\in R$ and suppose that the derivation of R that is effected by u is the restriction to R of a derivation of L that annihilates S. Then $[S, u]\subseteq Z(R)$. As an S-module, R is the direct sum of $Z(R)$ and a complementary submodule P. The component of u in P is annihilated by S so that $u\in R^S+Z(R)\subseteq Z(R)$. This means

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[1] Details will be found in G. F. Leger, *A note on the derivations of Lie algebras*, Proc. Amer. Math. Soc. vol. 4 (1953). We refer to this note as I.

[2] The author is grateful to the referee for putting Theorem 2 in this form.

that the derivation of R effected by u is 0. Now it follows from Theorem 1 that $D(L)$ splits over $I(L)$.

At the end of I is the following example (Hochschild's) of a nilpotent Lie algebra R such that $D(R)$ does not split over $I(R)$. R has a basis (x_1, x_2, x_3, x_4) over a ground field K (of characteristic 0) where (x_2, x_3, x_4) is abelian and $[x_1, x_2]=x_4$, $[x_1, x_3]=0$, $[x_1, x_4]=0$. Now let $L=S+R$ where S has a basis (s_1, s_2, s_3) over K with $[s_i, x_j]=0$, for $i=1, 2, 3$ and $j=3, 4$,

$$\begin{aligned} &[s_1, s_2] = 2s_2, && [s_2, s_3] = s_1, && [s_2, x_2] = x_1, \\ &[s_1, s_3] = -2s_3, && [s_1, x_2] = -x_2, && [s_3, x_1] = x_2, \\ &[s_1, x_1] = x_1, && [s_2, x_1] = 0, && [s_3, x_2] = 0. \end{aligned}$$

Then S is simple and $L=S+R$ is a Levi decomposition of L. Further $R^S \subseteq Z(R)$ so that $D(L)$ splits over $I(L)$ by Theorem 2. Thus we have an example of a Lie algebra L with radical R such that $D(L)$ splits over $I(L)$ but $D(R)$ does not split over $I(R)$.

This settles a question left open in I.

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