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A CORRECTION AND IMPROVEMENT OF A THEOREM ON ORDERED GROUPS

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In this note the notation and terminology of [1] will be used throughout. In particular, G will always denote an o-group with well ordered rank. Let P be the multiplicative group of positive rational numbers, and let R be the additive group of real numbers. In [1] the proofs of Theorems 2 and 3 are incorrect. This is a result of the careless formulation of the theorems by the author. Consider the following properties of G.

(1) Each component G^γ/G_γ of G has its group of o-automorphisms isomorphic to a subgroup P_γ of P.

(2) Each component G^γ/G_γ of G is o-isomorphic to a subgroup D_γ of R, and the only o-automorphisms of D_γ are multiplications by some elements of P.

(3) For each pair $\alpha \in \mathfrak{A}$ and $\gamma \in \Gamma$, there exists a pair m, n of positive integers such that $ng\alpha \equiv mg \bmod G_\gamma$ for all g in G^γ.

The statements of Theorems 2 and 3 include the hypothesis (1), but (2) and (3) are actually used in the proofs. Clearly (2) implies (1).

LEMMA. (a) (2) *is independent of the particular choice of* D_γ. (b) (2) *implies* (3). (c) (1) *does not imply* (2) *or* (3).

PROOF. (a) Let σ be an o-isomorphism of the subgroup A of R onto the subgroup B of R, and suppose that the only o-automorphisms of A are multiplications by some elements of P. If β is an o-automor-

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phism of B, then $\alpha=\sigma\beta\sigma^{-1}$ is an o-automorphism of A. Thus there exist positive integers m, n such that $na\alpha=ma$ for all a in A. Hence $nb\beta=[n(b\sigma^{-1})\alpha]\sigma=[m(b\sigma^{-1})]\sigma=mb$ for all b in B.

(b) Suppose that G satisfies (2), and consider $\alpha\in\mathfrak{A}$, $\gamma\in\Gamma$ and $g\in G^{\gamma}$. Then since the rank of G is well ordered, $G^{\gamma}\alpha=G^{\gamma}$. Thus α induces an o-automorphism α' of G^{γ}/G_{γ}. Let σ be an o-isomorphism of G^{γ}/G_{γ} onto a subgroup D_{γ} of R. Then $\tau=\sigma^{-1}\alpha'\sigma$ is an o-automorphism of D_{γ}. Thus there exist positive integers m, n such that $nd\tau=md$ for all d in D_{γ}.

$$\begin{aligned} G_{\gamma}+ng\alpha &= n(G_{\gamma}+g\alpha) = n(G_{\gamma}+g)\alpha' = [n((G_{\gamma}+g)\sigma)\tau]\sigma^{-1} \\ &= [m(G_{\gamma}+g)\sigma]\sigma^{-1} = G_{\gamma}+mg. \end{aligned}$$

(c) Let G be the naturally ordered rational vector space contained in R with basis $\{\pi^{n}: n \text{ is an integer}\}$, and let A be the multiplicative group generated by P and π. Then the o-automorphisms of G consist of multiplications by elements of A, hence G does not satisfy (2) or (3). But A is isomorphic to P because both are free abelian groups of countable rank, hence G satisfies (1).

Using Hahn's embedding theorem it is easy to show that if G is divisible and abelian, then (3) implies (2), but in general (3) appears to be weaker than (2). If (1) is replaced by (2) in Theorem 2, and (1) is replaced by (3) in Theorem 3, then the proofs given in [1] are correct. Moreover, *the proof of Theorem* 3 *does not make use of the hypothesis that G is abelian.* Thus we have a theorem about non-abelian o-groups.

THEOREM. *If G is an o-group with well ordered rank that satisfies* (3), *then the group* $\mathfrak{A}$ *of all o-automorphisms of G can be ordered. Moreover,* $\mathfrak{A}$ *can be ordered so that the group* $\mathfrak{B}$ (*of all o-automorphisms of G that induce the identity automorphism on each component*) *is a convex subgroup of* $\mathfrak{A}$.

PROOF. By the proof of Theorem 3, $\mathfrak{A}$ can be ordered. This induces an ordering of $\mathfrak{B}$ so that if $\beta\in\mathfrak{B}$ is positive, then $\alpha^{-1}\beta\alpha$ is positive for all α in $\mathfrak{A}$. Thus to prove the last part of the theorem, it suffices to show that $\mathfrak{A}/\mathfrak{B}$ can be ordered. Let Q be the large direct product of the groups of o-automorphisms of the components of G. Then Q is a torsion free abelian group, and hence it can be ordered. Each α in $\mathfrak{A}$ induces an o-automorphism α_{γ} of G^{γ}/G_{γ}. The mapping of α upon $(\cdots, \alpha_{\gamma}, \cdots)$ in Q is a homomorphism of $\mathfrak{A}$ into Q with kernel $\mathfrak{B}$. Thus $\mathfrak{A}/\mathfrak{B}$ is isomorphic to a subgroup of Q.

COROLLARY I. *If G is an o-group with well ordered rank that satisfies* (2), *then* $\mathfrak{A}$ *can be ordered*

This is an immediate consequence of property (b) of our lemma. Let A be a subgroup of R that contains 1. It would strengthen this corollary somewhat if one could find a property of A that is equivalent to the property that the only o-automorphisms of A consist of multiplications by elements of P. Or equivalently, $aA=A$ implies that $a\in P$ for all $0<a$ in R. A sufficient condition is that for each irrational number a in A, there exists a positive integer $n\neq 0$ such that $a^n\notin A$, but this condition is not necessary.

COROLLARY II. *If G is an o-group with well ordered rank, then the group of all o-automorphisms of G that satisfy condition* (3) *can be ordered.*

Finally let $\mathfrak{C}$ be a group of o-automorphisms of G that can be ordered, and let $H=\mathfrak{C}\times G$. For (α, a) and (β, b) in H we define $(\alpha, a)+(\beta, b)=(\alpha\beta, a\beta+b)$. Then H is a splitting extension of G by $\mathfrak{C}$. Define (α, a) positive if α is positive in $\mathfrak{C}$ or α is the identity automorphism and a is positive in G. Then H is an o-group.

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