EACH HOMOGENEOUS NONDEGENERATE CHAINABLE CONTINUUM IS A PSEUDO-ARC

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The endeavor to find all homogeneous plane continua continues. The simple closed curve and the point are obvious examples. The discovery of the pseudo-arc [1; 6] should have exploded (but did not) the conjectures that there are no others. A history of the problem with a discussion of various false starts is given in [4]. Finding the circle of pseudo-arcs [4] raised the number of known examples to four. Are there others as yet undiscovered? Jones showed [5] that each one which does not separate the plane is indecomposable. The theorem in this paper narrows the field for search still further.

We recall the following definitions:

A set X is *homogeneous* if for each pair of points p, q of X there is a homeomorphism of X onto itself that takes p onto q.

A continuum is *nondegenerate* if it contains more than one point.

An *ϵ-chain* is a finite ordered collection $d_1, d_2, \cdots, d_n$ of open sets, each of diameter less than ϵ, such that d_i intersects d_j if and only if i and j are adjacent integers.

A *snakelike* or *chainable* continuum is a compact metric continuum M such that for each positive number ϵ, M can be covered by an ϵ-chain.

A point p is an *end point* of a snakelike continuum M if for each positive number ϵ there is an ϵ-chain covering M such that the first link of the chain contains p.

A continuum is *indecomposable* if it is not the sum of two proper subcontinua. It is *hereditarily indecomposable* if each subcontinuum of it is indecomposable.

A *pseudo-arc* is a nondegenerate, hereditarily indecomposable, chainable continuum. Any two such continua are homeomorphic [2].

THEOREM. *Each homogeneous, nondegenerate, chainable continuum is a pseudo-arc.*

PROOF. First we show that M has an end point p. For each integer n, let q_n be a point of M such that a $1/n$-chain covers M and an end link of this chain contains q_n. Some subsequence of $q_1, q_2, \cdots$ converges to a point q. Then the point q of M has the following property:

Presented to the Society, February 23, 1957; received by the editors October 16, 1958.

[1] Work on this paper was supported by Contract NSF G1219.

Property of q. For each neighborhood N of q and each positive number ϵ there is an ϵ-chain covering M one of whose end links intersects M and lies in N. It follows from the homogeneity of M and the fact that each homeomorphism of M onto itself is uniformly continuous that each point of M has the *Property of q*.

Let d_1 be an end link of a 1-chain covering M such that d_1 contains a point p_1 of M. Since p_1 has the *Property of q*, there is an end link d_2 of a 1/2-chain covering M such that d_1 contains $\bar{d}_2$ and d_2 contains a point p_2 of M. Also, there is an end link d_3 of a 1/3-chain covering M such that d_2 contains $\bar{d}_3$ and d_3 contains a point p_3 of M. Similarly, we obtain $d_4, d_5, \cdots$. Then the point p which is the intersection of $d_1, d_2, \cdots$ is an end point of M.

Finally we show that M is hereditarily indecomposable. Assume M contains a continuum H which is the sum of two proper subcontinua H', H''. Let p be a point of $H' \cdot H''$. Then it follows from the homogeneity of M that p is an end point of M. However, as noted in [3], this would imply that one of H', H'' contains the other and this is impossible since each is a proper subcontinuum of their sum.

References

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