\[
\sum_{k=1}^{\infty} a_k \xi_k \sum_{k=1}^{\infty} \frac{1}{a_k} \xi_k \leq \frac{(M + m)^2}{4MM} \left[ \sum_{k=1}^{\infty} \xi_k^2 \right]^2
\]

for all \(x \in l_2\). This is inequality (2).

**References**


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**ON A PROBLEM OF LOHWATER**

G. PIRANIAN

Bagemihl [1] has shown that, for each complex-valued function \(f\) defined in the unit disk \(D\), there exist at most countably many points \(e^{i\theta}\) that are endpoints of two Jordan arcs \(\gamma_1(\theta)\) and \(\gamma_2(\theta)\) such that the corresponding cluster sets \(C(f, e^{i\theta}; \gamma_1(\theta))\) and \(C(f, e^{i\theta}; \gamma_2(\theta))\) are disjoint. Lohwater [2, p. 173] has recently asked whether there exists a function \(f\) for which uncountably many points \(e^{i\theta}\) are endpoints of three Jordan arcs \(\gamma_1(\theta), \gamma_2(\theta), \gamma_3(\theta)\) such that the intersection

\[
C(f, e^{i\theta}; \gamma_1(\theta)) \cap C(f, e^{i\theta}; \gamma_2(\theta)) \cap C(f, e^{i\theta}; \gamma_3(\theta))
\]

is empty. The property of points just described will be called the three-path property.

**Theorem.** There exists a complex-valued function, continuous in the unit disk, for which each point \(e^{i\theta}\) has the three-path property.

My proof is based on a slight modification of a technique recently used in connection with a problem on ambiguous points of a function defined in the unit sphere [3]. Let \(T_1, T_2, T_3\) be three trees in \(D\), with the property that each point \(e^{i\theta}\) can be approached along three Jordan arcs which lie on \(T_1, T_2, T_3\), respectively, except for their common endpoint \(e^{i\theta}\). Let \(T_1, T_2, T_3\) have the further property that no point

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lies on all three of the trees, and that the set of points lying on more than one of the trees and in the disk $|z| < r$ is finite, for $r < 1$.

Let $\omega = e^{2i\pi/3}$, and let $f(z) = \omega^{h+k}$ at all points of intersection of $T_h$ and $T_k$ ($h, k = 1, 2, 3; h \neq k$). Clearly, $f$ can be defined on the three trees in such a way that it is continuous on each of them, and so that $f(z) = e^{i\phi(z)}$ with

$$0 \leq \phi(z) \leq 2\pi/3 \quad \text{for } z \text{ on } T_1,$$

$$4\pi/3 \leq \phi(z) \leq 2\pi \quad \text{for } z \text{ on } T_2,$$

$$2\pi/3 \leq \phi(z) \leq 4\pi/3 \quad \text{for } z \text{ on } T_3.$$

The continuous extension of $f(z)$ into the remainder of the open unit disk presents no difficulty, and the proof is complete.

Added in Proof. A forthcoming paper by F. Bagemihl, G. Piranian and G. S. Young will exhibit a bounded holomorphic function for which uncountably many points $e^{i\theta}$ have the three-arc property.

References