

$$\sum_{k=1}^{\infty} a_k \xi_k^2 \sum_{k=1}^{\infty} \frac{1}{a_k} \xi_k^2 \leq \frac{(M+m)^2}{4mM} \left[\sum_{k=1}^{\infty} \xi_k^2 \right]^2$$

for all $x \in l_2$. This is inequality (2).

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ON A PROBLEM OF LOHWATER

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Bagemihl [1] has shown that, for each complex-valued function f defined in the unit disk D , there exist at most countably many points $e^{i\theta}$ that are endpoints of two Jordan arcs $\gamma_1(\theta)$ and $\gamma_2(\theta)$ such that the corresponding cluster sets $C(f, e^{i\theta}; \gamma_1(\theta))$ and $C(f, e^{i\theta}; \gamma_2(\theta))$ are disjoint. Lohwater [2, p. 173] has recently asked whether there exists a function f for which uncountably many points $e^{i\theta}$ are endpoints of three Jordan arcs $\gamma_1(\theta)$, $\gamma_2(\theta)$, $\gamma_3(\theta)$ such that the intersection

$$C(f, e^{i\theta}; \gamma_1(\theta)) \cap C(f, e^{i\theta}; \gamma_2(\theta)) \cap C(f, e^{i\theta}; \gamma_3(\theta))$$

is empty. The property of points just described will be called the *three-path property*.

THEOREM. *There exists a complex-valued function, continuous in the unit disk, for which each point $e^{i\theta}$ has the three-path property.*

My proof is based on a slight modification of a technique recently used in connection with a problem on ambiguous points of a function defined in the unit sphere [3]. Let T_1, T_2, T_3 be three trees in D , with the property that each point $e^{i\theta}$ can be approached along three Jordan arcs which lie on T_1, T_2, T_3 , respectively, except for their common endpoint $e^{i\theta}$. Let T_1, T_2, T_3 have the further property that no point

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lies on all three of the trees, and that the set of points lying on more than one of the trees and in the disk $|z| < r$ is finite, for $r < 1$.

Let $\omega = e^{2i\pi/3}$, and let $f(z) = \omega^{h+k}$ at all points of intersection of T_h and T_k ($h, k = 1, 2, 3; h \neq k$). Clearly, f can be defined on the three trees in such a way that it is continuous on each of them, and so that $f(z) = e^{i\phi(z)}$ with

$$\begin{aligned} 0 &\leq \phi(z) \leq 2\pi/3 && \text{for } z \text{ on } T_1, \\ 4\pi/3 &\leq \phi(z) \leq 2\pi && \text{for } z \text{ on } T_2, \\ 2\pi/3 &\leq \phi(z) \leq 4\pi/3 && \text{for } z \text{ on } T_3. \end{aligned}$$

The continuous extension of $f(z)$ into the remainder of the open unit disk presents no difficulty, and the proof is complete.

Added in Proof. A forthcoming paper by F. Bagemihl, G. Piranian and G. S. Young will exhibit a bounded holomorphic function for which uncountably many points $e^{i\theta}$ have the three-arc property.

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