$$H^*(\Omega G_2, Z) \cong Z[x, h(x), t(x)] \otimes Z\langle y\rangle$$

where $h(x) = \infty$ and $t(x)$ is defined by the greatest divisors $g(x^n) = n!/g(u^n)$. In particular, $g(x^2) = 1$ so that (1.15) fails for x^2 and $H^*(\Omega G_2, Z)$ has no system of divided powers.

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ANOTHER CUTPOINT THEOREM FOR PLANE CONTINUA

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If the subcontinuum M of a topological 2-sphere S does not separate S and is *locally connected*, then each pair of points of M, which are not separated in M by a point of M, belongs to the closure of a connected domain (of S) lying in M. This is true because each such pair of points belongs to a simple closed curve J lying in M and one of the complementary domains of J is a subset of M. However, without local connectedness such a simple closed curve may fail to exist. In fact, the proposition would then be false because (to take an extreme case) of the existence of indecomposable subcontinua of S which fail to separate S. While no point of an indecomposable continuum *separates* it, every point of it *cuts* it. Recently I showed [1] that this stronger form of separation (or rather the lack of it) is sufficient to restore the validity of the above proposition in the absence of local connectedness if a certain restriction were placed upon

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the *pair* of points under consideration. It is the purpose of this paper to remove even this restriction.

NOTATION AND TERMINOLOGY. If p and q are points of a continuum M and x is a point of $M-(p+q)$, x is said to *cut* p from q in M if *every* subcontinuum of M which contains $p+q$ contains x. By an *interior point* of M is meant a nonboundary point of M. By "the plane" is meant the Euclidean number plane with d denoting the usual Pythagorean distance function.

THEOREM. *Suppose that M is a compact subcontinuum of the plane S which does not separate S. If no point of M cuts the point p from the point q in M then some component of the set of interior points of M contains both p and q in its closure.*

INDICATION OF PROOF. If either p or q is an interior point of M, the theorem follows from a previous result [1]. So we have left to prove the theorem for the case when both p and q are boundary points of M.

Suppose that ϵ is a positive number such that $2\epsilon<d(p, q)$. Let $C_p(\epsilon)$ and $C_q(\epsilon)$ denote circles of radius ϵ centered on p and q respectively. There exists a simple domain $I(\epsilon)$ which contains M such that if $J(\epsilon)$ denotes the boundary of $I(\epsilon)$, y is a boundary point of M and z is a point of $I(\epsilon)+J(\epsilon)$ then $d[y, J(\epsilon)]<\epsilon$ and $d(z, M)<\epsilon$. There exist arcs $T_p(\epsilon)$ and $T_q(\epsilon)$ in $C_p(\epsilon)$ and $C_q(\epsilon)$ respectively such that each is minimal with respect to separating p from q in $I(\epsilon)+J(\epsilon)$ and $T_p(\epsilon)$ separates p from $q+T_q(\epsilon)$ in $I(\epsilon)+J(\epsilon)$. It follows that $T_q(\epsilon)$ separates $p+T_p(\epsilon)$ from q in $I(\epsilon)+J(\epsilon)$.

Since $T_p(\epsilon)$ and $T_q(\epsilon)$ have only their endpoints in $J(\epsilon)$, there exist in $J(\epsilon)$ two nonintersecting arcs $A(\epsilon)$ and $B(\epsilon)$ such that $T_p(\epsilon)+A(\epsilon)+T_q(\epsilon)+B(\epsilon)$ is a simple closed curve $H(\epsilon)$. Let $D(\epsilon)$ denote the bounded complementary domain of $H(\epsilon)$. If z is a point of $D(\epsilon)+H(\epsilon)$, then $d(z, M)<\epsilon$. Any subcontinuum of M which contains $p+q$ contains a subcontinuum irreducible from $T_p(\epsilon)$ to $T_q(\epsilon)$ which lies in $T_p(\epsilon)+D(\epsilon)+T_q(\epsilon)$.

Now let $L(\epsilon)$ denote a continuum lying in $T_p(\epsilon)+D(\epsilon)+T_q(\epsilon)$ which intersects both $T_p(\epsilon)$ and $T_q(\epsilon)$ such that if z belongs to $L(\epsilon)$, then $d[z, A(\epsilon)]=d[z, B(\epsilon)]$. There exists a simple infinite sequence α of values of ϵ such that $D(\epsilon)+H(\epsilon)$ converges to a subset of M and $L(\epsilon)\to L$ as $\epsilon\to 0$ in α. The set L has the following properties:

(a) L is a continuum containing both p and q,

(b) L is a subset of M, and

(c) every point of $L-(p+q)$ is an interior point of M. Properties (a) and (b) are evident. So it remains only to prove property (c).

Let x be a point of $L-(p+q)$. Since x does not cut p from q in

M, there exists a subcontinuum K of M which contains $p+q$ but not x. Let δ be a positive number such that $4\delta=d(x, K)$ and let $U_\delta(x)$ and $U_{3\delta}(x)$ be the circular regions centered on x of radius δ and 3δ respectively. When ϵ (in α) is sufficiently small $[T_p(\epsilon)+T_q(\epsilon)]\cdot[U_{3\delta}(x)]=0$ but $L(\epsilon)\cdot U_\delta(x)\neq 0$. Let y be some point of $L(\epsilon)\cdot U_\delta(x)$, let $r=\delta+d(x, y)$ and let $U_r(y)$ be a circular region of radius r and center y. Obviously $U_{3\delta}(x)\supset U_r(y)\supset U_\delta(x)$. So $[T_p(\epsilon)+T_q(\epsilon)]\cdot U_r(y)=0$. If $A(\epsilon)\cdot U_r(y)\neq 0$, let f be a point of $A(\epsilon)\cdot U_r(y)$ such that $d(f, y)=d[y, A(\epsilon)]$. But y belongs to $L(\epsilon)$. Hence there exists in $U_r(y)$ a point g of $B(\epsilon)$ such that $d(g, y)=d[g, B(\epsilon)]=d(f, y)$. The sum of the straight line intervals from y to f and from y to g is an arc T_y lying in $U_r(y)$, having only its endpoints f and g in $H(\epsilon)$, and containing the point y of $D(\epsilon)$. Hence $T_y-(f+g)\subset D(\epsilon)$. But $T_y\cdot K=0$ and K contains a continuum lying in $T_p(\epsilon)+D(\epsilon)+T_q(\epsilon)$ irreducible from $T_p(\epsilon)$ to $T_q(\epsilon)$. Since the points f and g separate $T_p(\epsilon)$ from $T_q(\epsilon)$ in $H(\epsilon)$ this involves a contradiction [2, Theorem 17, p. 167]. Hence $U_r(y)\cdot H(\epsilon)=0$ and since y belongs to $D(\epsilon)$, $U_r(y)\subset D(\epsilon)$; so for sufficiently small values of ϵ (in α), $U_\delta(x)\subset D(\epsilon)$. Consequently $U_\delta(x)$ is a subset of M and x is an interior point of M.

The continuum L contains a subcontinuum N which is irreducible from p to q; $N-(p+q)$ is connected and each of its points is an interior point of M. The component of the set of interior points of M which contains $N-(p+q)$ has both p and q in its closure.

COUNTEREXAMPLE. The converse of the theorem is false. Let L be the closure of the graph of $y=\sin 1/x$ $(-\pi\leqq x\leqq\pi)$ together with one arc (the lower one) of the square whose vertices are $(\pm\pi, \pm\pi)$ so that this arc joins the endpoints of the graph, and let M denote L together with its bounded complementary domain D. Obviously $\overline{D}=M$ but $(0, -1)$ cuts $(0, -\pi)$ from $(0, 1)$ in M.

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