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A REMARK ON THE CHARACTERIZATION OF HOMOTHETIC TRANSFORMATION AND INVERSION[1]

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Given two closed orientable C^2 surfaces S, $\bar{S}$ in E^3, let $h: S \to \bar{S}$ be a differentiable homeomorphism, such that referring to a suitable origin O

$$\bar{X} = kX, \tag{1}$$

with k nonzero and C^2 at points at which $X \neq 0$, $\bar{X} \neq 0$. Write $p = -X \cdot N$, $\bar{p} = -\bar{X} \cdot \bar{N}$. Suppose further that S, $\bar{S}$ contains no pieces of cones with vertex O. We use the form $b = (\bar{N} \times X) \cdot dX$, introduced in [1]. We generalize results of [1] to

Theorem 1′. *If* $\iint_S (H \mp k\bar{H}) p\, dA = 0$, *then* $k = \text{const.}$, *i.e. the map* h *is a homothetic transformation with center O.*

Proof. The formulae $\bar{p}\, d\bar{A} = k^3 p\, dA$ and $db = (2/k^2)\bar{p}\bar{H}\, d\bar{A} + 2\bar{N} \cdot N\, dA$ yield $\iint_S N \cdot \bar{N}\, dA = -\iint_S k\bar{H} p\, dA$. Combining with $\iint_S dA = \iint_S H p\, dA$ gives

$$\iint_S (1 \mp N \cdot \bar{N})\, dA = \iint_S (H \pm k\bar{H}) p\, dA,$$

and the theorem follows as in [1].

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THEOREM 3'. *If* $\iint_S(H\pm k(\bar{H}+2\bar{X}\cdot\bar{N}/\bar{X}\cdot\bar{X}))dA=0$ *and* $O\notin S$, $\bar{S}$ *then h is an inversion with center O.*

PROOF. This follows from Theorem 1' as Theorem 3 in [1] followed from Theorem 1.

COROLLARY 1. *Given a closed orientable* C^2 *surface S and an interior point O such that the reciprocal of the Gaussian curvature at each point equals its square distance from O. Then S is a sphere with center O.*

PROOF. Taking O as origin we have

$$1/K = 1/X\cdot X$$

and then

$$\iint_S (H + X\cdot N/X\cdot X)dA = \iint_S (H - pK)dA = 0,$$

that is

$$\iint_S \{H + (H + 2X\cdot N/X\cdot X)\}dA = 0.$$

By Theorem 3' the identity map on S is an inversion. Therefore S is a sphere.

COROLLARY 2. *Given a closed orientable* C^2 *surface S, at each point P on S define* θ *as the angle between OP and the outward normal at P. If* $OP=(1/H)\cos\theta$, *then S is a sphere with center O.*

PROOF. Taking O as origin our condition becomes

$$H + X\cdot N/X\cdot X = 0.$$

Then the proof is the same as in Corollary 1.

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