REMARK. If X_1, X_2 are two continuous curves and $\rho_i: X_i \to I, i=1, 2$, are monotone mappings onto, then $S(X_1, X_2, \rho_1, \rho_2)$ need not be locally connected.

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A CLAN WITH ZERO WITHOUT THE FIXED POINT PROPERTY

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There is a conjecture due to A. D. Wallace that a clan (i.e., a compact, connected, topological semigroup with identity element) with a zero element has the fixed point property. This is related to another conjecture of Wallace that a compact connected topological lattice has the fixed point property [4]. A proof of the latter conjecture for the finite dimensional case has recently been given by Dyer and Shields [1]. There is an example due to Kinoshita [2] of a contractable continuum without the fixed point property. The purpose of this note is to exhibit a multiplication which will make Kinoshita's example into a clan with zero, and, thus, provide a counter example to the first conjecture above.

We exhibit first a result which seems to be rather generally known, but which, to the author's knowledge, does not appear in print.

LEMMA. *Suppose S is a topological semigroup, and f is an open or closed map taking S onto T, a Hausdorff space. Suppose further that $f(a)=f(b)$ and $f(c)=f(d)$ implies $f(ac)=f(bd)$. Then T can be given a multiplication which makes it a topological semigroup and which makes f a homomorphism.*

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PROOF. For t_1 and t_2 in T we define $t_1 \cdot t_2$ as follows. Let a_1 and a_2 be elements of S such that $f(a_1)=t_1$ and $f(a_2)=t_2$. Let $t_1 \cdot t_2 = f(a_1 a_2)$. It is easily seen that the multiplication is well defined and associative, and that f is a homomorphism. The only item which remains to be checked is the continuity of the multiplication. Let m: $T \times T \to T$ be defined by $m(t_1, t_2) = t_1 \cdot t_2$. We need only show that m is continuous. If f is an open map, let P be an open set in T (if f were closed, we would, of course, take P closed). Then $m^{-1}(P) = \{(t_1, t_2) \mid t_1 \cdot t_2 \in P\} = \{(t_1, t_2) \mid f[f^{-1}(t_1) f^{-1}(t_2)] \in P\}$ is seen to be open since f and multiplication in S are continuous and f is open. Thus m is continuous completing the proof.

Now let

$$A = \{(r, \theta, z) \mid 0 \leqq r < 1, z = 0\},$$

$$B = \left\{(r, \theta, z) \mid r = \frac{2}{\pi} \tan^{-1} \theta, \theta \geqq 0, 0 \leqq z \leqq 1\right\},$$

and

$$C = \{(r, \theta, z) \mid r = 1, 0 \leqq z \leqq 1\},$$

where r, θ, and z represent the usual cylindrical coordinates in three space. Let $K = A \cup B \cup C$, then K is the continuum of the example of Kinoshita. For a continuous function on K to K which moves every point, the reader is referred to Kinoshita [**2**].

Let D be the projection of $B \cup C$ into the $z=0$ plane. Define a multiplication "o" on D by

$$(r_1, \theta_1) \circ (r_2, \theta_2) = \left[\max\left\{r_1, r_2, \frac{2}{\pi} \tan^{-1}(\theta_1 + \theta_2)\right\}, \theta_1 + \theta_2\right].$$

It is easy to see that "o" is associative and continuous and that (0, 0) is an identity element. It, perhaps, should be mentioned that D, in slightly different form, is a rather well known clan (see e.g. the example on page 286 of [**3**]).

Now let I be the interval [0, 1] with the usual multiplication, and let $E = D \times I$ with the coordinatewise multiplication; i.e., $(r_1, \theta_1, z_1) \cdot (r_2, \theta_2, z_2) = [(r_1, 0_1) \circ (r_2, 0_2), z_1 z_2]$.

Let f: $E \to R^3$ be defined by

$$f(r, \theta, z) = (r, \theta, z) \text{ if } r \leqq 2z$$

and

$$f(r, \theta, z) = (2z, \theta, z) \text{ if } r \geqq 2z.$$

Let $F=f(E)$. It is easily seen that f is continuous and, since E is compact, that f is closed. Let p_i be the points (r_i, θ_i, z_i), $i=1, 2, 3, 4$, and suppose $f(p_1)=f(p_2)$ and $f(p_3)=f(p_4)$. We want to show $f(p_1\ p_3)=f(p_2\ p_4)$. If $p_1=p_2$ and $p_3=p_4$, the result is clear. Hence suppose, say, $p_1\neq p_2$. Then $f(p_1)=f(p_2)$ implies $r_1\geqq 2z_1$, $r_2\geqq 2z_2$, $\theta_1=\theta_2$, and $z_1=z_2$. Now the r coordinate of $p_1p_3\geqq r_1\geqq 2z_1\geqq 2z_1z_3$ and similarly the r coordinate of $p_2p_4 \geqq 2z_2z_4$. Hence $f(p_1p_3) = (2z_1z_3,\ \theta_1+\theta_3,\ z_1z_3) = (2z_2z_4,\ \theta_2+\theta_4,\ z_2z_4)=f(p_2p_4)$. Thus there is induced on F the multiplication described in the lemma. With respect to this multiplication the points (0, 0, 1) and (0, 0, 0) are respectively the identity element and zero of F. Moreover it is clear that F is homeomorphic to K. For example, the function h defined by $h(r,\theta,z)=(r,\theta,(1-r/2)z+r/2)$ takes K homeomorphically onto F. Applying the lemma again, h^{-1} induces a multiplication which makes K a clan with zero as was to be shown.

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