be given ($r \geqq s$; $(a_1', \cdots, a_s')$ may be the null set). Then (2.4) and (2.5) associate with (7.1)

$$(7.2) \qquad (p_1, \cdots, p_n) \quad \text{and} \quad (p_1', \cdots, p_n')$$

respectively. Since the primary set (7.1) are distinct, there must be an a_i such that $a_i \notin \{a_1', \cdots, a_s'\}$. The n-tuplets (7.2) will be distinct since $p_{a_i} \neq p'_{a_i}$. Therefore the lattice points associated with the primary sets (7.1) will be distinct.

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SUBGROUPS OF THE UNIMODULAR GROUP[1]

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Following the notation of [3], we let Γ denote the proper unimodular group consisting of all 2×2 matrices with rational integral elements and determinant $+1$. For m a positive integer, define the *principal congruence group* $\Gamma(m)$ by

$$(1) \qquad \Gamma(m) = \{X \in \Gamma: X \equiv I \pmod{m}\},$$

where I denotes the identity matrix in Γ, and where congruence of matrices is interpreted as elementwise congruence.

For p a prime, we know from [2] that $\Gamma(p)$ is a free group with a finite set S of generators. If we define

$$(2) \qquad T_m = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix},$$

then S may be chosen to include T_p. For each fixed integer s, we may define a group $\Omega(p, s)$ consisting of all power products of the generators in S for which the exponent sum for each generator is a multiple of s. In [3] it was shown that each $\Omega(p, s)$ is a normal subgroup of Γ of finite index in Γ. Furthermore, if $s > 1$ and $(s, p) = 1$, it was proved that $\Omega(p, s)$ does not contain any principal congruence group.

Let $\Delta(m)$ denote the normal subgroup of Γ which is generated by T_m. Obviously $\Delta(m) \subset \Gamma(m)$. Recently, Brenner [1] raised the following questions:

A. Does $\Delta(m) = \Gamma(m)$ for all m?

Received by the editors May 2, 1960.

[1] This research was supported by the Office of Naval Research.

B. For each m does there exist a positive integer k such that $\Delta(m) \supset \Gamma(mk)$?

The purpose of this note is to show that the answer to both questions is "No."

THEOREM. *If $m>1$ and m is not a power of a prime, then $\Delta(m)$ does not contain any principal congruence group.*

PROOF. From the hypothesis we may write

$$m = p^r s, \qquad r \geqq 1, \qquad s > 1, \qquad (p, s) = 1.$$

Since T_m is a power of T_{ps}, we have $\Delta(m) \subset \Delta(ps)$. Further, $T_{ps} = T_p^s$ implies that $T_{ps} \in \Omega(p, s)$. But $\Omega(p, s)$ is a normal subgroup of Γ, and so we conclude

$$\Delta(m) \subset \Delta(ps) \subset \Omega(p, s). \tag{3}$$

Since $\Omega(p, s)$ contains no principal congruence group, the same holds for $\Delta(m)$. Q.E.D.

Brenner showed that $\Delta(m) = \Gamma(m)$ for $1 \leqq m \leqq 5$. The above theorem implies that $\Delta(6)$ contains no principal congruence group. We are thus left with the following problem: For prime power values of m, can $\Delta(m)$ contain a principal congruence group?

The only additional light we can shed on this problem comes from Frasch [2], who showed:

Let $p \geqq 7$, p prime. Then $\Delta(p)$ is properly contained in $\Gamma(p)$.

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