$=1+2x+i(x-y)$, then $m=1$, $a=1$, $c=2$. To describe its "canonical form" F_1 satisfying (4), it is convenient to introduce the function $K(x, y)=((8x+1)/(8y+1))^{1/2}$. A simple calculation shows that $F_1(x, y)=2\exp(-i\log K)(K^{1/q}+i(K^{1/q}-K^{-1/p}))$.

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THE NONEXISTENCE OF PROJECTIONS FROM L^1 TO H^1

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Consider the Banach space $L^1(0, 2\pi)$ and the subspace H^1, of all functions all of whose negative Fourier coefficients vanish. It has been conjectured that H^1 has no complement in L^1, i.e., that L^1 is not the direct sum of H^1 and some other Banach space. In this note we give a proof of this conjecture.

The conjecture is of course equivalent to the following statement on projection operators.

THEOREM. *There exists no bounded linear operator* $P: L^1 \to H^1$ *for which* $Pf=f$ *for all* $f\in H^1$.

PROOF. Suppose such a P existed. Let $l_n(f)$ denote the nth Fourier coefficient of $P(f)$; then l_n is a bounded linear functional on L^1 and as a result we have

$$l_n(f(\theta)) = \int_0^{2\pi} f(\theta)\phi_n(\theta)d\theta,$$

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where $\phi_n(\theta)$ is a bounded function. Also we know that

$$l_n(e^{ni\theta}) = 1 \ \text{(assuming } n \geqq 0)$$

and so

$$\int_0^{2\pi} |\phi_n(\theta)| \, d\theta \geqq 1.$$

It follows that

$$\int_0^{2\pi} \sum_{n=1}^{N} \frac{|\phi_n(\theta)|}{n} d\theta \geqq \log N, \qquad N > 1, N \text{ fixed}.$$

So that, for some θ_0, $\sum_{n=1}^N |\phi_n(\theta_0)|/n \geqq (1/2\pi) \log N$. Thus choosing $\epsilon_n = \text{sg}\, \bar{\phi}_n(\theta_0)$ we are assured that $\text{Max}_\theta \left| \sum_{n=1}^N \epsilon_n \phi_n(\theta)/n \right| \geqq (1/2\pi) \log N$ (the inequality holding in fact for θ_0).

We can therefore determine a function $f(\theta)$ with $\int_0^{2\pi} |f| \leqq 1$ such that

$$\text{(1)} \qquad \int_0^{2\pi} f(\theta) \sum_{n=1}^{N} \frac{\epsilon_n \phi_n(\theta)}{n} d\theta \geqq \frac{1}{10} \log N$$

but

$$\left| \int_0^{2\pi} f(\theta) \sum_{n=1}^{N} \frac{\epsilon_n \phi_n(\theta)}{n} d\theta \right| \leqq \sum_{n=1}^{N} \frac{|l_n(f)|}{n} = \sum_{n=1}^{N} \frac{|a_n|}{n},$$

where $\sum a_n e^{in\theta}$ is the Fourier series for Pf. Since $Pf \in H^1$, however, by Hardy's theorem [1],

$$\text{(2)} \qquad \sum_{n=1}^{N} \frac{|a_n|}{n} \leqq 2\pi \int |Pf| \leqq 2\pi \|P\|.$$

Finally combining (1) and (2) gives the contradiction

$$\frac{1}{10} \log N \leqq 2\pi \|P\|, \qquad \text{for all } N > 1,$$

and this completes the proof.

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