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ON PROJECTIVE MODULES OVER SEMI-HEREDITARY RINGS

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This note contains a proof of the following

THEOREM. *Each projective module P over a (one-sided) semi-hereditary ring Λ is a direct sum of modules, each of which is isomorphic with a finitely generated ideal of Λ.*

This theorem, already known for finitely generated projective modules [**1**, I, Proposition 6.1], has been recently proved for arbitrary projective modules over commutative semi-hereditary rings by I. Kaplansky [**2**], who raised the problem of extending it to the non-commutative case.

We recall two results due to Kaplansky:

Any projective module (over an arbitrary ring) is a direct sum of countably generated modules [**2**, Theorem 1].

If any direct summand N of a countably generated module M is such that each element of N is contained in a finitely generated direct summand, then M is a direct sum of finitely generated modules [**2**, Lemma 1].

According to these results, it is sufficient to prove the following proposition:

Each element of the module P is contained in a finitely generated direct summand of P.

Let $F=P\oplus Q$ be a free module and x be an arbitrary element of P. Let $x=\lambda_1x_1+\cdots+\lambda_nx_n$ be a representation of the element x in some base for the free module F and let G denote the free submodule

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of F, generated by the elements $x_1, \cdots, x_n$. Consider the homomorphism $\phi: G \rightarrow Q$, induced by the projection of F onto Q. Obviously, Ker $\phi = G \cap P \ni x$ and $G/G \cap P$ is isomorphic to Im ϕ. Since Im ϕ is a finitely generated submodule of the projective module Q and the ring Λ is semi-hereditary, Im ϕ and therefore $G/G \cap P$ is projective. It follows that $G \cap P$ is a direct summand of G and hence finitely generated. Since G is a direct summand of F, the module $G \cap P$ is also a direct summand of F and, being contained in P, a direct summand of P.

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