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TULANE UNIVERSITY AND
ST. MARY'S DOMINICAN COLLEGE

IMMERSIONS OF ALMOST PARALLELIZABLE MANIFOLDS

MORRIS W. HIRSCH[1]

The purpose of this note is to prove the following theorem:

An almost parallelizable n-manifold M can be immersed in Euclidean q-space R^q *if* $2q>3n$.

By *immersion* $f: M\to R^p$ we mean a continuously differentiable map whose Jacobian matrix has rank $n=\dim M$ at each point. We denote the *differential* of an immersion f by df.

A *regular homotopy* $f_t: M\to R^n$ is a homotopy such that each f_t is an immersion and df_t is a homotopy of the tangent bundle of M into R^n. In this case f_0 and f_1 have equivalent normal bundles.

We say M is *almost parallelizable* if the tangent bundle of $M-x$ is trivial, for some $x\in M$.

To prove the theorem, we first observe that if M is not compact, or is bounded, then M is parallelizable, and by [1, 6.3], M can be immersed in $R^{n+1}\subset R^q$. Hence we assume M is compact and unbounded. Let B be an n-ball diffeomorphically embedded in M, with bounding $(n-1)$ sphere S. Put $M_0=M-\text{int } B$. By the remark above, there is an immersion f: $M_0\to R^{n+1}$. We consider f as an immersion in R^q, and we deform f through a regular homotopy near S, keeping $f|S$ fixed, so that if X is a unit tangent vector to M at point $x\in S$ pointing into M_0, then $df(X)$ is the unit vector $e=(0, \cdots, 0, 1)$ normal to R^{q-1} in R^q. We still have $f(S)\subset R^{q-1}$.

Since the immersion f is regularly homotopic to an immersion $M\to R^{n+1}$, the normal bundle of f is trivial. This enables us to apply a lemma [2, 3.2] of M. Kervaire, which implies that the *Smale in-*

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variant of $f|S$ vanishes. By [**3**, E], therefore, there exists an immersion $g: B \rightarrow R^{q-1}$ such that $g|S = f|S$. We consider g as an immersion in R^q, and we deform g through a regular homotopy, so that if X is the vector above, $dg(-X) = -e$. We now define $h: M \rightarrow R^q$ by $h(x) = f(x)$ or $h(x)$, according to whether $x \in M_0$ or $x \in B$. It is clear that h is an immersion, and the theorem is proved.

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University of California, Berkeley