Baire spaces cannot be removed. If $Q$ is the space of rationals in $E_1$ with the relative topology, there is a separately continuous $f: Q \times Q \to E_1$ which is zero on a dense subset of $Q \times Q$ but not identically zero.

**Reference**


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**COMPLETE SEQUENCES OF FUNCTIONS**

CASPER GOFFMAN

Although the result of this note is implicitly contained in the work of A. A. Talalyan [2] and could also have been a corollary to the theorem in [1], it seems to be of sufficient interest to merit explicit treatment.

It is known (see [1]) that if $\{f_1, f_2, \ldots, f_n, \ldots\}$ is a sequence of measurable functions which is complete in the space $M$ of measurable functions (i.e., every measurable $f$ is the limit in measure of a sequence of finite linear combinations of $\{f_1, f_2, \ldots, f_n, \ldots\}$) then $\{f_n, f_n, \ldots, f_n, \ldots\}$ is also complete in $M$.

Let $X$ be a separable Banach space of measurable functions on $[a, b]$ such that for every measurable $G \subseteq [a, b]$, with $m(G)>0$, the set $X_G$ of restrictions of the functions in $X$ to $G$ is a Banach space and

(a) If $\{g_n\}$ converges to $g$ in $X$ then $\{g_n\}$ converges to $g$ in $X_G$,

(b) The set of bounded measurable functions is a dense subset of $X$; hence, of $X_G$, for every $G$,

(c) For every $G$, uniform convergence on $G$ implies convergence in $X_G$ and convergence in $X_G$ implies convergence in measure on $G$.

**Theorem.** If $\{f_1, f_2, \ldots, f_n, \ldots\}$ is complete in $X$ and $\epsilon>0$, there is a measurable $G \subseteq [a, b]$, with $m(G)>\ (b-a)-\epsilon$, such that $\{f_2, f_2, \ldots, f_n, \ldots\}$ is complete in $X_G$.

**Proof.** Let $\{g_1, g_2, \ldots, g_n, \ldots\}$ be dense in $X$. Since $\{f_1, f_2, \ldots, f_n, \ldots\}$ is complete in $X$, it follows from (b), (c) and

Received by the editors July 18, 1961.

1 Supported by National Science Foundation Grant NSF-G18920.
the fact that the bounded functions are dense in $M$, that
\[ \{f_1, f_2, \ldots, f_m, \ldots\} \] is complete in $M$. It follows from [1] that
for every $n$, there is a sequence \( \{\phi_1, \phi_2, \ldots, \phi_m, \ldots\} \) of finite linear
combinations of \( \{f_2, f_3, \ldots, f_n, \ldots\} \) which converges in measure to
\( g_n \), and so has a subsequence \( \{\psi_1, \psi_2, \ldots, \psi_m, \ldots\} \) which converges
uniformly to \( g_n \) on a measurable set \( G_n \), with \( m(G_n) > (b-a) - \epsilon/2^n \).
Let \( G = \bigcap_{n=1}^{\infty} G_n \). Since uniform convergence on \( G \) implies convergence
in \( X_\sigma \) by (a), and since \( \{g_1, g_2, \ldots, g_n, \ldots\} \) is dense in \( X_\sigma \) by (c),
it follows that \( \{f_2, f_3, \ldots, f_n, \ldots\} \) is complete in \( X_\sigma \).

If \( X = L_2[a, b] \), then \( X_\sigma = L_2(G) \), so that we have:

**Corollary.** If \( \{f_1, f_2, \ldots, f_n, \ldots\} \) is complete for \( L_2[a, b] \) and
\( \epsilon > 0 \) there is a measurable \( G \subset [a, b] \), \( m(G) > (b-a) - \epsilon \) such that
\( \{f_2, f_3, \ldots, f_n, \ldots\} \) is complete for \( L_2(G) \).

**References**

1. C. Goffman and D. Waterman, *Basic sequences in the space of measurable func-
2. A. A. Talalyan, *Representing of measurable functions by series*, Uspehi Mat.

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