# A NOTE ON THE GENUS OF A KNOT

S. KINOSHITA[1]

For each polygonal knot $k$ in a 3-sphere $S^3$ there exists a polyhedral orientable surface $M_k$ whose boundary is $k$ ([1; 2]). The genus of such a knot $k$, denoted by $g(k)$ below, was defined by H. Seifert [2] as the minimal number of genera of all such $M_k$. Now let us define $\tilde{g}(k)$ for each polygonal knot $k$ as the minimal number of genera of all orientable surfaces $M_k$ whose boundaries are $k$, where $M_k$ may be wildly imbedded. Is $\tilde{g}(k)=g(k)$? This is a problem proposed by R. H. Fox. The purpose of this short note is to prove that *the equality holds*.

PROOF. Let $k$ be a polygonal knot in $S^3$. Let $M$ be a polyhedral orientable surface and $h$ a homeomorphism of $M$ in $S^3$ such that $h(\partial M)=k$. Then we are only to prove that $g(M)$, the genus of $M$, is equal to or greater than $g(k)$.

Let $T$ be a tubular neighborhood of $k$. Bing's approximation theorem [3] shows that for each $\epsilon>0$ there exists a semilinear homeomorphism $h'$ of $M$ into $S^3$ such that for each $x\in M$ $d(h(x), h'(x))<\epsilon$. If we choose $\epsilon$ as to be sufficiently small, we may suppose that $h'(\partial M)=k'$ is contained in $T$ and that $k$ is a companion of $k'$ with the winding number one in the sense of H. Schubert [4]. By definition $g(h'(M))\geqq g(k')$. Further H. Schubert [4] shows that if $k$ is a companion of $k''$ with the winding number $\alpha$, then $g(k'')\geqq \alpha g(k)+g(k^*)$, where $k^*$ is a suitably defined polygonal knot. Thus $g(k')\geqq g(k)$ in our case. Therefore $g(M)=g(h'(M))\geqq g(k')\geqq g(k)$, which completes the proof.

## REFERENCES

1. F. Frankl and L. Pontrjagin, *Ein Knotensatz mit Anwendung auf die Dimensionstheorie*, Math. Ann. **102** (1930), 785–789.

2. H. Seifert, *Über das Geschlecht von Knoten*, Math. Ann. **110** (1934), 571–592.

3. R. H. Bing, *An alternative proof that 3-manifolds can be triangulated*, Ann. of Math. **69** (1959), 37–65.

4. H. Schubert, *Knoten und Vollringe*, Acta Math. **90** (1953), 131–286.

INSTITUTE FOR ADVANCED STUDY
AND OSAKA UNIVERSITY

Received by the editors May 1, 1961.

[1] Supported by National Science Foundation grant G14779.