We conclude with an example of a space $X \subset C_c(R)$ (a dense subspace of $C_0(R)$, by Theorem 3) which contains a nontrivial nonnegative function and on which a positive measure (not a Haar measure) acts invariantly: X consists of all $f \in C_c(R)$ with $\hat{f}(1) = \hat{f}(-1) = 0$, and $d\mu(x) = (2 + \sin x)dx$. Since $2i \sin x = e^{ix} - e^{-ix}$, $\int_{-\infty}^{\infty} f(x) \sin x dx = 0$ for all $f \in X$, and so μ acts invariantly on X; also, X contains the nonnegative triangular function f defined by

$$f(x) = \max(2\pi - |x|, 0) \qquad (-\infty < x < \infty). \tag{27}$$

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BARRELLED SPACES AND THE OPEN MAPPING THEOREM[1]

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1. **Introduction.** If E and F are any two topological vector spaces then the following statement may or may not be true:

(A) If f is any linear and continuous mapping of E onto F then f is open.

It is well known [1] that (A) is true when E and F are Fréchet spaces. An extension due to Pták [6], and Robertson and Robertson [7] is that (A) is true if E is B-complete and F is barrelled (t-space). We ask here whether these results characterize Fréchet and B-complete spaces respectively. More precisely, let $\mathfrak{F}$ and $\mathfrak{I}$ denote the classes of all Fréchet and barrelled spaces respectively. We ask if a topological vector space E, having the property that (A) is true whenever $F \in \mathfrak{F}(\mathfrak{I})$, is necessarily a Fréchet (B-complete) space.

A well-known example of an LF-space and a theorem of Dieudonné and Schwartz [5, Theorem 1] supplies a counterexample to the above for $\mathfrak{F}$. Here, we give an example showing that the other case is also false.

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We use the following definitions and notations:

A linear subspace Q of the dual E' of a locally convex space E is said to be *almost closed* if and only if, for each neighborhood U of 0 in E, $Q \cap U^0$ is $\sigma(E', E)$-closed in the relative $\sigma(E', E)$-topology of U^0 (U^0 is the polar of U).

A locally convex space E is said to be *B-complete* if and only if every almost closed subspace of E' is $\sigma(E', E)$-closed.

Let E' be the dual of a locally convex space E. E'^{σ} and E'^{τ} will denote the point set E' endowed with the topology $\sigma(E', E)$ and $\tau(E', E)$ respectively.

2. **The example.** Let $E = L_1(N)$ be the space of all absolutely convergent sequences with the topology induced from R^N (the countable product of the reals). Since E is dense in R^N it is not B-complete. The dual of E, E', is $R^{(N)}$ and $\tau(E', E)$ is a norm topology with $\|x\| = \operatorname{Sup}_n |x_n|$.

Let $f: E \to F$ be a linear and continuous mapping of E onto a barrelled space F. Then in the dual $f': F'^{\sigma} \to E'^{\sigma}$ will be a homeomorphism into. We will identify F'^{σ} with its image as a subspace of E'^{σ}. Since F is barrelled, F'^{σ} is a quasi-complete subspace of E'^{σ}; therefore, it will be quasi-complete in E'^{τ}. But E'^{τ} is normed hence F'^{τ} is a Banach space. Therefore F' is closed. This proves that f is open.

We actually have proven more. Since F' is a subspace of E'^{τ} it has a finite or countable Hamel basis. Hence F' is finite-dimensional [**3**, (9), p. 37]. This proves that the only barrelled spaces F which are continuous images of E are the finite-dimensional ones.

Remark. A detailed study of spaces satisfying (A) when $F \in \mathfrak{I}$, has been carried out by one of us [**4**].

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