AN INEQUALITY FOR NONNEGATIVE ENTIRE FUNCTIONS

R. P. BOAS, JR.

I give a simple proof of an inequality equivalent to one that was proved by S. Bernstein [1]. The proof also applies in higher dimensions, where Bernstein's method is not available.

**Theorem 1.** Let \( f(z) \) be an entire function of exponential type \( \tau \), nonnegative and integrable on the real axis. Then

\[
f(z) \leq (2\pi)^{-1} \int_{-\infty}^{\infty} f(x) \, dx.
\]

There is equality for \( f(z) = z^{-2} \sin^2 \tau z/2 \).

We have (see, e.g., [2, p. 103])

\[
f(z) = \int_{-\tau}^{\tau} \phi(t) e^{itz} \, dt.
\]

A special case of Poisson's summation formula,\(^2\) applied to (2), yields

\[
\tau \phi(0) = \sum_{n=-\infty}^{\infty} f((2n\pi + x)/\tau).
\]

Since the terms on the right are nonnegative, none of them can exceed \( \tau \phi(0) \), which is the right-hand side of (1).

Now let \( f(z, w) \) be an entire function of exponential type, absolutely integrable for real \( z, w \). We then have [5]

\[
f(x, y) = \int_{S} \int \phi(t, u) e^{itz + iu} \, dt \, du,
\]

where \( S \) is a bounded convex set determined by the growth of \( f(z, w) \). Consider lattices \( \{a_{11}m + a_{12}n, a_{12}m + a_{22}n\} \) such that \( S \) is inside a lattice parallelogram that contains \( (0, 0) \) and has area \( 4(a_{11}a_{22} - a_{12}a_{21}) = 4 \det[a] \); let \( 4D \) be the area of the smallest such parallelogram.

**Theorem 2.** If \( f(x, y) \geq 0 \) then

---

Received by the editors March 21, 1962.

1 This was written while the author was President's Fellow at Northwestern University.

2 The abstract form of Poisson's formula is given in [4, p. 153]; the special cases used here can be found in [3].
If $[A]$ is the matrix $2\pi[a]^{-1}$ the analogue of (3) is

$$D\phi(0, 0) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(A_{11}m + A_{12}n + x, A_{21}m + A_{22}n + y),$$

and (5) follows in the same way as (1).

REFERENCES


REPRESENTATIONS OF BANACH SPACES

G. S. YOUNG

Banach and Mazur proved that every separable Banach space $B$ can be represented as the space $C(M)$ of continuous real functions on a compact metric space $M$. Since $M$ is the continuous image of the Cantor set $K$, $C(M)$ can be imbedded in $C(K)$, and since functions in $C(K)$ can be extended preserving norm to functions over $I$, they conclude that $B$ can be represented as a subspace of $C(I)$.

If $B$ is not separable, it can be represented as $C(H)$, where $H$ is compact Hausdorff. A compact Hausdorff space is the continuous image of some totally disconnected compact Hausdorff space $T$—for example, give the space the discrete topology, and let $T$ be its Stone-Čech compactification. It follows that $B$ is isomorphic to a subspace of $C(T)$. If $T$ could be given a linear order inducing the same topology, we could fill in the missing intervals and obtain a compact con-