Since \( \int_0^x |F| < \infty \), the first equation in (4) has a solution, say \( \theta_1 \), such that \( \theta_1 \to 0 \) as \( x \to \infty \). Put \( r_1 = \exp -\int_0^x F \cos^2 (f + \theta) \). Then \( r_1 \) and \( \theta_1 \) constitute a solution of (4). Set \( y_1 = r_1 \sin (f + \theta) \). One can construct \( y_1 \) similarly.

**REFERENCES**


**REMARK ON \( \pi(x) = o(x) \)**

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In Hardy and Wright [1], Landau [2], and Prachar [3] there are proofs of \( \pi(x) = o(x) \). The purpose of this paper is to provide a simpler proof.

**THEOREM.** \( \pi(x) = o(x) \).

**Proof.** Let \( M_r \) denote the product of the first \( r \) primes. Given any positive real number \( x \geq 2 \), it is clear that there exist unique positive integers \( k \) and \( r \), with \( 1 \leq k \leq p_{r+1} - 1 \), such that

\[
(1) \quad k \cdot M_r \leq x < (k + 1)M_r.
\]

For any \( x \) satisfying (1) we must also have

\[
(2) \quad \pi(x) \leq (k + 1)\phi(M_r) + r
\]

(where \( \phi(M_r) \) is the totient of \( M_r \)). From (1) and (2) we obtain

\[
(3) \quad 0 \leq \frac{\pi(x)}{x} \leq \frac{(k + 1)\phi(M_r)}{kM_r} + \frac{r}{kM_r}.
\]

Since \( kM_r \geq kr! \geq r! \) and \( (k + 1)/k \leq 2 \), we replace (3) by

\[
(4) \quad 0 \leq \frac{\pi(x)}{x} \leq 2 \prod_{i=1}^r \left( 1 - \frac{1}{p_i} \right) + \frac{1}{(r - 1)!}.
\]
Since there are infinitely many primes (a fact already used implicitly in defining \( k \) and \( r \)), \( r \to \infty \) as \( x \to \infty \). We may therefore write

\[
0 \leq \limsup_{x \to \infty} \frac{\pi(x)}{x} \leq 2 \prod_{i=1}^{\infty} (1 - \frac{1}{p_i}) + \lim_{r \to \infty} \frac{1}{(r-1)!} = 0,
\]

where the divergence of \( \prod_{i=1}^{\infty} (1 - \frac{1}{p_i}) \) to zero follows from the divergence of \( \sum_{i=1}^{\infty} \frac{1}{p_i} \).

**References**