THE ACTION OF Γ_{2n} ON $(n-1)$-CONNECTED $2n$-MANIFOLDS

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This note makes the results of [1] more precise in certain cases. We assume the notations of that paper.

THEOREM. *If* $n \equiv 3, 5, 6, 7 \pmod 8$, *$T$ is a $2n$-sphere representing $x \in \Gamma_{2n}$, M is a closed $(n-1)$-connected $2n$-manifold, and if there is an orientation preserving diffeomorphism h of $M \# T$ on M, then $x=0$.*

It is known that Γ_m may be interpreted both as the quotient $\mathrm{Diff}(S^{m-1})/i^* \mathrm{Diff}(D^m)$ and as the group of differential structures on S^m $(m \neq 4)$. The two interpretations are connected, for given a diffeomorphism f of S^{m-1}, representing x, we may glue two copies of D^m using it, and derive a differential structure on S^m. We shall reformulate this. We may suppose without loss of generality that f keeps a disc D^{m-1} fixed. Form a manifold L from $S^{m-1} \times I$ by identifying each $(P, 1)$ with $(fP, 0)$. Then L contains $D^{m-1} \times S^1$. Make a spherical modification, replacing this by $S^{m-2} \times D^2$.

LEMMA. *The resulting manifold is a sphere, representing x.*

PROOF. We cut the whole figure in half, cutting I at 0 and 1/2. L falls into two pieces, of which one is $(S^{m-1} \times S^1_+)$, where the modification replaces $D^{m-1} \times S^1_+$ by $S^{m-2} \times D^2_+$ (the subscript $+$ indicates that the second coordinate is non-negative). This yields a disc; similarly for the other half. These are now to be glued by a diffeomorphism of the boundary which is the identity except on $D^{m-1} \times 1$, where it agrees with f. Thus it is equivalent to f. Hence we get a sphere, representing x.

We now prove the theorem. The diffeomorphism h may be supposed fixed on a disc, and then induces a diffeomorphism g of its complement N (whose boundary is S^{2n-1}) on itself. Here we are thinking of the disc as used to form the connected sum $M \# T$, and so $g | \partial N = f$ represents x. Form V from $N \times I$ by identifying $(P, 1)$ with $(gP, 0)$. Again we may suppose f fixed on a disc, and so $D^{2n-1} \times S^1$ contained in ∂V. Form W by attaching along it $D^{2n-1} \times D^2$. By the lemma, ∂W represents x. It is at once verified that W is $(n-1)$-connected with M, and in view of our hypothesis on n, it follows that

Received by the editors January 8, 1962.

W is n-parallelisable. Now by the main theorem of [2], ∂W bounds a contractible manifold, and so represents the zero element of Γ_{2n}.

References

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2. ———, *Killing the middle homotopy group of odd dimensional manifolds*, Trans. Amer. Math. Soc. **103** (1962), 421–433.

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THE COEFFICIENTS IN THE EXPANSION OF CERTAIN PRODUCTS

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1. The identities

$$(1)\qquad \prod_{n=0}^{\infty}(1-p^n x)^{-1}=\sum_{n=0}^{\infty}\frac{x^n}{(1-p)(1-p^2)\cdots(1-p^n)},$$

$$(2)\qquad \prod_{n=0}^{\infty}(1-p^n x)=\sum_{n=0}^{\infty}\frac{(-1)^n p^{n(n-1)/2}x^n}{(1-p)(1-p^2)\cdots(1-p^n)},$$

where $|p|<1$, are well known. The more general products

$$\prod_{m,n=0}^{\infty}(1-p^m q^n x)^{-1},\qquad \prod_{m,n=0}^{\infty}(1-p^m q^n x)\qquad (|p|<1,\ |q|<1)$$

have been discussed in [1; 2].

In the present note we consider the products

$$(3)\quad \prod_{n=0}^{\infty}(1-p^n x-q^n y)^{-1},\ \prod_{n=0}^{\infty}(1-p^n x-p^n y)\ (|p|<1,\ |q|<1).$$

Put

$$(4)\qquad F(x,y)=\prod_{n=0}^{\infty}(1-p^n x-q^n y)^{-1}=\sum_{r,s=0}^{\infty}A_{rs}x^r y^s,$$

where $A_{rs}=A_{rs}(p,q)$ is independent of x and y. It follows from (4) that

Presented to the Society, November 10, 1961; received by the editors November 7, 1961.

[1] Supported in part by National Science Foundation grant G-16485.