# THE ACTION OF $\Gamma_{2n}$ ON $(n-1)$-CONNECTED $2n$-MANIFOLDS

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This note makes the results of [1] more precise in certain cases. We assume the notations of that paper.

THEOREM. *If $n \equiv 3, 5, 6, 7 \pmod 8$, $T$ is a $2n$-sphere representing $x \in \Gamma_{2n}$, $M$ is a closed $(n-1)$-connected $2n$-manifold, and if there is an orientation preserving diffeomorphism $h$ of $M \# T$ on $M$, then $x=0$.*

It is known that $\Gamma_m$ may be interpreted both as the quotient $\mathrm{Diff}(S^{m-1})/i^* \mathrm{Diff}(D^m)$ and as the group of differential structures on $S^m$ $(m \neq 4)$. The two interpretations are connected, for given a diffeomorphism $f$ of $S^{m-1}$, representing $x$, we may glue two copies of $D^m$ using it, and derive a differential structure on $S^m$. We shall reformulate this. We may suppose without loss of generality that $f$ keeps a disc $D^{m-1}$ fixed. Form a manifold $L$ from $S^{m-1} \times I$ by identifying each $(P, 1)$ with $(fP, 0)$. Then $L$ contains $D^{m-1} \times S^1$. Make a spherical modification, replacing this by $S^{m-2} \times D^2$.

LEMMA. *The resulting manifold is a sphere, representing $x$.*

PROOF. We cut the whole figure in half, cutting $I$ at 0 and $1/2$. $L$ falls into two pieces, of which one is $(S^{m-1} \times S^1_+)$, where the modification replaces $D^{m-1} \times S^1_+$ by $S^{m-2} \times D^2_+$ (the subscript $+$ indicates that the second coordinate is non-negative). This yields a disc; similarly for the other half. These are now to be glued by a diffeomorphism of the boundary which is the identity except on $D^{m-1} \times 1$, where it agrees with $f$. Thus it is equivalent to $f$. Hence we get a sphere, representing $x$.

We now prove the theorem. The diffeomorphism $h$ may be supposed fixed on a disc, and then induces a diffeomorphism $g$ of its complement $N$ (whose boundary is $S^{2n-1}$) on itself. Here we are thinking of the disc as used to form the connected sum $M \# T$, and so $g|\partial N = f$ represents $x$. Form $V$ from $N \times I$ by identifying $(P, 1)$ with $(gP, 0)$. Again we may suppose $f$ fixed on a disc, and so $D^{2n-1} \times S^1$ contained in $\partial V$. Form $W$ by attaching along it $D^{2n-1} \times D^2$. By the lemma, $\partial W$ represents $x$. It is at once verified that $W$ is $(n-1)$-connected with $M$, and in view of our hypothesis on $n$, it follows that

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$W$ is $n$-parallelisable. Now by the main theorem of [2], $\partial W$ bounds a contractible manifold, and so represents the zero element of $\Gamma_{2n}$.


REFERENCES

1. C. T. C. Wall, *Classification of $(n-1)$-connected $2n$-manifolds*, Ann. of Math. **75** (1962), 163–189.

2. ———, *Killing the middle homotopy group of odd dimensional manifolds*, Trans. Amer. Math. Soc. **103** (1962), 421–433.


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# THE COEFFICIENTS IN THE EXPANSION OF CERTAIN PRODUCTS

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1. The identities

$$\prod_{n=0}^{\infty}(1-p^n x)^{-1}=\sum_{n=0}^{\infty}\frac{x^n}{(1-p)(1-p^2)\cdots(1-p^n)}, \tag{1}$$

$$\prod_{n=0}^{\infty}(1-p^n x)=\sum_{n=0}^{\infty}\frac{(-1)^n p^{n(n-1)/2}x^n}{(1-p)(1-p^2)\cdots(1-p^n)}, \tag{2}$$

where $|p|<1$, are well known. The more general products

$$\prod_{m,n=0}^{\infty}(1-p^m q^n x)^{-1}, \qquad \prod_{m,n=0}^{\infty}(1-p^m q^n x) \qquad (|p|<1,\ |q|<1)$$

have been discussed in [1; 2].

In the present note we consider the products

$$\prod_{n=0}^{\infty}(1-p^n x-q^n y)^{-1},\ \prod_{n=0}^{\infty}(1-p^n x-p^n y)\ (|p|<1,\ |q|<1). \tag{3}$$

Put

$$F(x,y)=\prod_{n=0}^{\infty}(1-p^n x-q^n y)^{-1}=\sum_{r,s=0}^{\infty}A_{rs}x^r y^s, \tag{4}$$

where $A_{rs}=A_{rs}(p,q)$ is independent of $x$ and $y$. It follows from (4) that

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