A NOTE ON PERMUTATIONS IN AN ARBITRARY FIELD

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The writer [1] has proved that every permutation on the numbers of the finite field $GF(q)$ is generated by the special permutations

$$x^{q-2}, \quad \alpha x + \beta \qquad (\alpha, \beta \in GF(q), \alpha \neq 0). \tag{1}$$

Let F denote an arbitrary field. Define the function

$$x^* = \begin{cases} x^{-1} & (x \in F, x \neq 0), \\ 0 & (x = 0). \end{cases} \tag{2}$$

Clearly x^* defines a permutation of F.

The following theorem holds.

THEOREM 1. *Every transposition* $(\alpha\beta)$, *where* $\alpha, \beta \in F$ *is finitely generated by the special permutations*

$$x^*, \quad \gamma x + \delta \qquad (\gamma, \delta \in F, \gamma \neq 0). \tag{3}$$

The proof (compare [1]) follows from consideration of the function

$$g(x) = -\alpha^2\left(\left((x - \alpha)^* + \frac{1}{\alpha}\right)^* - \alpha\right)^*,$$

where α is a fixed nonzero number of F. It is easily verified that $g(x)$ represents the transposition (0α).

Let $G = G(F)$ denote the group consisting of all finite products

$$t_1 t_2 \cdots t_n,$$

where the t_j are arbitrary transpositions $(\alpha\beta)$. As an immediate corollary of Theorem 1 we have

THEOREM 2. *The group G is generated by the special permutations* (3).

REFERENCE

1. L. Carlitz, *Permutations in a finite field*, Proc. Amer. Math. Soc. 4 (1953), 538.

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