A NOTE ON THE HOMOLOGY OF DELETED PRODUCT SPACES

C. W. PATTY

If X is a topological space, let D_X denote the subset of $X \times X$ consisting of the set of all points of the form (x, x), where $x \in X$. Then the deleted product space, X^*, of X is the space $X \times X - D_X$ with the relative topology. The purpose of this note is to prove the following theorem.

THEOREM. *Let Z denote the group of integers. If X is a finite, n-dimensional polyhedron such that $H_n(X, Z) = 0$, then $H_{2n}(X^*, Z) = 0$.*

If X is a finite polyhedron, let

$$P(X^*) = \bigcup \{ r \times s \mid r \text{ and } s \text{ are simplexes of } X \text{ and } r \cap s = \emptyset \}.$$

It is shown in [1] that if X is a finite polyhedron, then there is a deformation retraction of X^* onto $P(X^*)$. If X is a finite polyhedron, let $X' = \bigcup \{ r \times s \mid r \text{ and } s \text{ are faces of simplexes } r' \text{ and } s' \text{ of } X \text{ such that } r' \cap s' \neq \emptyset \}$.

LEMMA. *If X is a finite, n-dimensional polyhedron, then $P(X^*) \cap X'$ has dimension $\leqq 2n-1$.*

PROOF. Let σ be a cell of $P(X^*) \cap X'$. Since σ is a cell of $P(X^*)$, $\sigma = r \times s$, where r and s are simplexes of X such that $r \cap s = \emptyset$. Since σ is a cell of X', r and s are faces of simplexes r' and s' such that $r' \cap s' \neq \emptyset$. Therefore, either r is a proper face of r' or s is a proper face of s'. Hence $r \times s$ has dimension $\leqq 2n-1$.

PROOF OF THE THEOREM. Now $(X \times X; P(X^*), X')$ is a proper triad. Consider the Mayer-Vietoris sequence of this proper triad.

$$\leftarrow H_{2n}(P(X^*), Z) + H_{2n}(X', Z) \leftarrow H_{2n}(P(X^*) \cap X', Z) \leftarrow 0$$
$$\cdots \leftarrow H_{2n}(X \times X, Z).$$

By the lemma, $H_{2n}(P(X^*) \cap X', Z) = 0$, and $H_{2n}(X \times X, Z) = 0$ by the Künneth formula. Therefore, by exactness of the Mayer-Vietoris sequence, $H_{2n}(P(X^*), Z) = 0$.

REFERENCE

1. A. Shapiro, *Obstructions to the imbedding of a complex in a euclidean space.* I. *The first obstruction,* Ann. of Math. (2) **66** (1957), 256–269.

UNIVERSITY OF NORTH CAROLINA

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