A WILD SPHERE WHICH CAN BE PIERCED AT EACH
POINT BY A STRAIGHT LINE SEGMENT

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In this note we describe a wild 2-sphere \( S \) in \( E^3 \) which can be pierced at each point by a straight line segment. The existence of such a wild sphere answers the following question which was raised by R. H. Bing in his address Embedding surfaces in 3-manifolds at the 1962 International Congress of Mathematicians in Stockholm:

Is a 2-sphere in \( E^3 \) tame if it can be pierced at each point by a straight line interval?

In order to make the description of \( S \) as concise as possible, we make use of the terminology and notation of [2], the author's modification of Bing's well-known "Dog Bone Decomposition" (see [1]). We let \( G \) be the decomposition space defined in [2], and let \( A_0 \) be the union of the nondegenerate elements of \( G \). It is possible to carry out the construction of \( A_0 \) in such a way that the endpoints of the components of \( A_0 \) lie in two parallel planes, and we assume that this is the case. We can represent \( A_0 = \bigcap_{n=0}^{\infty} B_n \), where each set \( B_n \) is the union of \( 2^n \) admissible polyhedra (see [2, p. 502, Figure 1]) \( P_1^n, P_2^n, \ldots, P_{2^n} \). Now, if (in the notation of [2]) \( P_j^n \) is represented as \( L \cup M \cup R \), we define \( Q_j^n \) to be (basic parallelepiped of \( L \) \( \cup M \) \( \cup \) (basic parallelepiped of \( R \)). We now define \( S \) to be the boundary of

\[
\bigcup_{n=0}^{\infty} \bigcup_{j=1}^{2^n} Q_j^n.
\]

It is easy to see that \( S \) is a "horned" 2-sphere and hence is wildly imbedded in \( E^3 \). \( A_0 \) is contained in the union of \( S \) and the bounded component of \( E^3 - S \), and for each component \( C \) of \( A_0 \) the set \( C \cap S \) consists of the endpoints of \( C \).

If \( p \in A_0 \cap S \), then we can pierce \( S \) at \( p \) by extending the component \( C \) of \( A_0 \) which contains \( p \). On the other hand, if \( p \in S - A_0 \), then \( S \) is locally polyhedral at \( p \) and can certainly be pierced by a line segment at \( p \).

In view of the above example, R. H. Bing has raised the following two questions:

(1) Is a topological 2-sphere \( S \) in \( E^3 \) tame if corresponding to each point \( p \in S \) there are Euclidean (round) spheres \( \sigma_1 \) and \( \sigma_2 \) containing \( p \) such that \( \sigma_1 - p \) and \( \sigma_2 - p \) lie on opposite sides of \( S \)?

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(2) Is a topological 2-sphere \( S \) in \( E^3 \) tame if corresponding to each point \( p \in S \) there are cones \( \gamma_1 \) and \( \gamma_2 \), each with vertex at \( p \), such that \( \gamma_1 - p \) and \( \gamma_2 - p \) lie on opposite sides of \( S \)?

Bibliography

1. R. H. Bing, A decomposition of \( E^3 \) into points and tame arcs such that the decomposition space is topologically different from \( E^3 \), Ann. of Math. (2) 65 (1957), 484–500.


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CORRECTION TO “A CHARACTERIZATION OF QF-3 ALGEBRAS”

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J. P. Jans is kind enough to inform me a gap of Necessity proof in my paper appearing in these Proceedings, 13 (1962), 701–703. In this note I shall report Theorem 2 in the paper is however valid by a slight alteration of the proof. In p. 702, the argument between line 9 and line 18 should be replaced by the following: Let \( e_\lambda \) be a primitive idempotent of \( A \) such that \( l(N)e_\lambda \neq 0 \). Then there exists an element \( x \in L \) such that \( l(N)e_\lambda x \neq 0 \) for \( L \) is faithful. Denote \( x \) by \( \sum_{\alpha \in \Lambda} a_\alpha e_\alpha + a_\lambda e_\lambda \), \( a_\alpha, a_\lambda \in A \). Since \( e_\lambda (\sum_{\alpha \in \Lambda} a_\alpha e_\alpha) \subseteq N \), \( l(N)e_\lambda x = l(N)e_\lambda a_\lambda e_\lambda \) and we have \( l(N)e_\lambda L e_\lambda \neq 0 \). Here, suppose \( L e_\lambda \neq A e_\lambda \).

Then \( L e_\lambda \subseteq N e_\lambda \) for \( N e_\lambda \) is the unique maximal left ideal of \( A e_\lambda \) and it follows \( l(N)e_\lambda L e_\lambda \subseteq l(N)N = 0 \). But this is a contradiction. Thus we obtain \( L e_\lambda = A e_\lambda \). Now, let \( \theta \) be the epimorphism: \( L \rightarrow L e_\lambda (= A e_\lambda) \), defined by \( \theta(x) = xe_\lambda \) for all \( x \in L \). Since \( L e_\lambda \) is projective, we have a direct sum decomposition of \( L: L_\lambda \oplus L'_\lambda \), where \( L_\lambda \cong A e_\lambda \). Then as \( \text{Hom}(L, K) \) is monomorphic to \( P \) and \( \text{Hom}(A e_\lambda, K) \) is injective, \( \text{Hom}(A e_\lambda, K) \) is isomorphic to a direct summand of \( P \). Thus if we denote by \( \Lambda \) the set of all indices \( \lambda \) such that \( l(N)e_\lambda \neq 0 \), \( \text{Hom}(\sum_{\lambda \in \Lambda} A e_\lambda, K) \) is projective.