A WILD SPHERE WHICH CAN BE PIERCED AT EACH POINT BY A STRAIGHT LINE SEGMENT

M. K. FORT, JR.[1]

In this note we describe a wild 2-sphere S in E^3 which can be pierced at each point by a straight line segment. The existence of such a wild sphere answers the following question which was raised by R. H. Bing in his address *Embedding surfaces in* 3-*manifolds* at the 1962 International Congress of Mathematicians in Stockholm:

Is a 2-*sphere in* E^3 *tame if it can be pierced at each point by a straight line interval?*

In order to make the description of S as concise as possible, we make use of the terminology and notation of [2], the author's modification of Bing's well-known "Dog Bone Decomposition" (see [1]). We let G be the decomposition space defined in [2], and let A_0 be the union of the nondegenerate elements of G. It is possible to carry out the construction of A_0 in such a way that the endpoints of the components of A_0 lie in two parallel planes, and we assume that this is the case. We can represent $A_0=\bigcap_{n=0}^{\infty} B_n$, where each set B_n is the union of 2^n admissible polyhedra (see [2, p. 502, Figure 1]) $P_1^n, P_2^n, \cdots, P_{2^n}^n$. Now, if (in the notation of [2]) P_j^n is represented as $L\cup M\cup R$, we define Q_j^n to be (basic parallelepiped of L)$\cup M\cup$(basic parallelepiped of R). We now define S to be the boundary of

$$\bigcup_{n=0}^{\infty} \bigcup_{j=1}^{2^n} Q_j^n.$$

It is easy to see that S is a "horned" 2-sphere and hence is wildly imbedded in E^3. A_0 is contained in the union of S and the bounded component of E^3-S, and for each component C of A_0 the set $C\cap S$ consists of the endpoints of C.

If $p\in A_0\cap S$, then we can pierce S at p by extending the component C of A_0 which contains p. On the other hand, if $p\in S-A_0$, then S is locally polyhedral at p and can certainly be pierced by a line segment at p.

In view of the above example, R. H. Bing has raised the following two questions:

(1) Is a topological 2-sphere S in E^3 tame if corresponding to each point $p\in S$ there are Euclidean (round) spheres σ_1 and σ_2 containing p such that σ_1-p and σ_2-p lie on opposite sides of S?

Received by the editors September 8, 1962.

[1] This work was supported by NSF Grant G23790.

(2) Is a topological 2-sphere S in E^3 tame if corresponding to each point $p \in S$ there are cones γ_1 and γ_2, each with vertex at p, such that $\gamma_1 - p$ and $\gamma_2 - p$ lie on opposite sides of S?

Bibliography

1. R. H. Bing, *A decomposition of E^3 into points and tame arcs such that the decomposition space is topologically different from E^3*, Ann. of Math. (2) **65** (1957), 484–500.

2. M. K. Fort, Jr., *A note concerning a decomposition space defined by Bing*, Ann. of Math. (2) **65** (1957), 501–504.

University of Georgia

CORRECTION TO "A CHARACTERIZATION OF QF-3 ALGEBRAS"

HIROYUKI TACHIKAWA

J. P. Jans is kind enough to inform me a gap of Necessity proof in my paper appearing in these Proceedings, **13** (1962), 701–703. In this note I shall report Theorem 2 in the paper is however valid by a slight alteration of the proof. In p. 702, the argument between line 9 and line 18 should be replaced by the following: Let e_λ be a primitive idempotent of A such that $l(N)e_\lambda \neq 0$. Then there exists an element $x \in L$ such that $l(N)e_\lambda x \neq 0$ for L is faithful. Denote x by $\sum_{\kappa \neq \lambda} a_\kappa e_\kappa + a_\lambda e_\lambda$, $a_\kappa, a_\lambda \in A$. Since $e_\lambda(\sum_{\kappa \neq \lambda} a_\kappa e_\kappa) \subseteq N$, $l(N)e_\lambda x = l(N)e_\lambda a_\lambda e_\lambda$ and we have $l(N)e_\lambda L e_\lambda \neq 0$. Here, suppose $Le_\lambda \neq Ae_\lambda$. Then $Le_\lambda \subseteq Ne_\lambda$ for Ne_λ is the unique maximal left ideal of Ae_λ and it follows $l(N)e_\lambda L e_\lambda \subseteq l(N)N = 0$. But this is a contradiction. Thus we obtain $Le_\lambda = Ae_\lambda$. Now, let θ be the epimorphism: $L \to Le_\lambda (= Ae_\lambda)$, defined by $\theta(x) = xe_\lambda$ for all $x \in L$. Since Le_λ is projective, we have a direct sum decomposition of $L: L_\lambda \oplus L_\lambda'$, where $L_\lambda \approx Ae_\lambda$. Then as $\mathrm{Hom}(L, K)$ is monomorphic to P and $\mathrm{Hom}(Ae_\lambda, K)$ is injective, $\mathrm{Hom}(Ae_\lambda, K)$ is isomorphic to a direct summand of P. Thus if we denote by Λ the set of all indices λ such that $l(N)e_\lambda \neq 0$, $\mathrm{Hom}(\sum_{\lambda \in \Lambda} Ae_\lambda, K)$ is projective.