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A COVERING THEOREM FOR CONVEX MAPPINGS[1]

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The following theorems are classical. Proofs can be found in [1, pp. 214, 223].

Theorem 1. *If $f(z)$ is regular and univalent in $|z|<1$, $f(0)=0$ and $f'(0)=1$ then the image domain covers the circle $|w|<1/4$.*

Theorem 2. *If $f(z)$ is regular and univalent in $|z|<1$, $f(0)=0$, $f'(0)=1$ and the image D is convex, then D covers the circle $|w|<1/2$.*

The purpose of this note is to show that Theorem 2 can be proven as a simple consequence of Theorem 1. Suppose then that $f(z)$ satisfies the hypotheses of Theorem 2 and $f(z)\neq c$. Let $g(z)=(f(z)-c)^2$. Suppose that z_1 and z_2 are distinct points in the unit circle and $g(z_1)=g(z_2)$. Then either $f(z_1)=f(z_2)$ or $2^{-1}(f(z_1)+f(z_2))=c$. The first equation cannot hold since $f(z)$ is univalent in $|z|<1$. Neither can the second equation hold, for D is convex and therefore the average of every two points in D is also in D. This proves that $g(z)$ is univalent in $|z|<1$. The function $h(z)=(c^2-g(z))/2c$ satisfies the hypotheses of Theorem 1, and $h(z)\neq c/2$ since $f(z)\neq c$. Therefore, $|c/2|\geqq 1/4$, $|c|\geqq 1/2$.

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