# ON AN INEQUALITY OF OPIAL, BEESACK AND LEVINSON

R. N. PEDERSON[1]

Levinson [4] recently gave a simple proof of the inequality

$$\int_0^a |y(x)y'(x)|\ dx \leqq \frac{a}{2}\int_0^a |y'(x)|^2\, dx$$

valid for absolutely continuous complex-valued $y(x)$ satisfying $y(0)=0$. Equality holds only if $y=bx$ for some constant $b$. Various forms of the inequality had been proved in [1], [2], and [3].

In this note we give an even simpler proof. We first make the preliminary estimate

$$\int_0^a |y(x)y'(x)|\ dx \leqq \int_0^a \int_0^x |y'(t)y'(x)|\ dtdx.$$

Since the integral of a symmetric integrand over the triangle $0\leqq t\leqq x$, $0\leqq x\leqq a$, is equal to half of its integral over the square $0\leqq t\leqq a$, $0\leqq x\leqq a$, we easily obtain

$$\begin{aligned}\int_0^a \int_0^x |y'(t)y'(x)|\ dtdx &= \frac{1}{2}\int_0^a \int_0^a |y'(t)y'(x)|\ dtdx \\ &= \frac{1}{2}\left[\int_0^a |y'(x)|\ dx\right]^2.\end{aligned}$$

The conclusion then follows immediately from the Schwarz inequality.

## References

1. Paul R. Beesack, *On an integral inequality of Z. Opial*, Trans. Amer. Math. Soc. **104** (1962), 470–475.

2. C. Olech, *A simple proof of a certain result of Z. Opial*, Ann. Polon. Math. **8** (1960), 61–63.

3. Z. Opial, *Sur une inégalité*, Ann. Polon. Math. **8** (1960), 29–32.

4. N. Levinson, *On an inequality of Opial and Beesack*, Proc. Amer. Math. Soc. **15** (1964), 565–566.

Carnegie Institute of Technology

Received by the editors September 10, 1964.

[1] Supported by the National Science Foundation, NSF-G25060.