AN EXAMPLE IN ČECH COHOMOLOGY

DANIEL S. KAHN

In this note, we give an example of a compact space $X$ with integral Čech cohomology groups $H^g(X) = 0$, $g > 0$, but which can be mapped essentially onto the three-sphere $S^3$. This cannot occur for finite-dimensional $X$ [2].

We construct such an $X$ for each odd prime $p$, which we now suppose fixed. Define $X_0$ to be $S^{2p} \cup \mathbb{E}^{3p+1}$, the $2p$-sphere with a $(2p+1)$-cell attached by a map of degree $p$. Inductively, we define $X_{n+1}$ to be the $(2p-2)$-fold suspension $E^{2p-2}X_n$ of $X_n$, $n \geq 0$. We also define maps $\alpha_n : X_n \to X_{n-1}$, $n > 1$, by $\alpha_n = E^{2p-2}\alpha_{n-1}$, where $\alpha_1$ is defined as follows: Let $\beta : S^{2p-1} \to S^3$ represent a generator of $\pi_{2p}(S^3; p) \approx \mathbb{Z}_p$. Then $E^{2p-2}\beta : S^{4p-2} \to S^{2p}$ admits a coextension $\beta' : S^{4p-2} \to S^{3p} \cup \mathbb{E}^{2p+1}$ [4, p. 13]. Since the homotopy class of $\beta'$ is of order $p$, $\beta'$ admits an extension $\alpha_1 : S^{4p-2} \cup \mathbb{E}^{4p-1} \to S^{4p-1} \cup \mathbb{E}^{2p+1}$. We note that $\beta$ admits an extension $\alpha : S^{2p} \cup \mathbb{E}^{2p+1} \to S^3$. We now define $X = \text{Inv Lim} \{X_n, \alpha_n\}$.

It is evident that $H^g(X) = 0$, $g > 0$. The composites $f_n = \alpha_1 \alpha_2 \cdots \alpha_n : X_n \to S^3$ define a map $f : X \to S^3$. The proof that $f$ is essential depends on the following result of Toda [5], [1].

**Theorem [Toda].** Each $f_n$ is an essential map. Further, all suspensions of $f_n$ are essential.

Since $[X, S^3]$, the set of homotopy classes of maps of $X \to S^3$, is equal to $\text{Dir Lim} \{[X_n, S^3], \alpha_{n+1}\}$ [3, p. 228], $f$ is essential.

$X$ has the further property that $E^{n(2p-2)}X = X$, $n > 0$. The theorem of Toda implies that each $E^{n(2p-2)}f : E^{n(2p-2)}X = X \to S^{3+n(2p-2)}$ is also essential.

**Bibliography**


THE UNIVERSITY OF CHICAGO

Received by the editors May 15, 1964.

1 This research was partially supported by National Science Foundation Grant GP-623.

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