A NOTE ON ADDITION CHAINS

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A sequence of integers $1 = a_0 < a_1 \cdots < a_r = n$, is called an addition chain for $n$, if $a_i = a_j + a_k$ for $1 \leq i \leq r; 0 \leq j, k < i$. For a given $n$, the least $r$ for which such a chain exists is called $l(n)$.

Scholz [3] conjectured:

$$l(2^q - 1) \leq 1(q) + q - 1, \quad q \geq 1.$$  

A. Brauer [1] proved (1), provided there is a minimal chain $\{a_i\}_{i=0}^{l(q)}$ for $q$ such that $a_i = a_{i-1} + a_i, 0 < i \leq l(q), 0 \leq t \leq i - 1$. Gioia, Subbarao, and Sugunamma [2] employ eight lemmas to prove (1) if:

$$q = 2^{c_1} + 2^{c_2} + 2^{c_3} \quad c_1 > c_2 > c_3 \geq 0.$$  

Lemma 4 of [2] states that, if (2) holds, $l(q) = c_1 + 2$. It is observed here that (1), subject to (2) follows immediately from this lemma and Brauer's result, since

$$1, 2, 4, \cdots, 2^{c_1}, 2^{c_1} + 2^{c_3}, 2^{c_1} + 2^{c_3} + 2^{c_2}$$

is a minimal chain for $q$ which satisfies Brauer's condition.

REFERENCES


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