A NOTE ON ADDITION CHAINS

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A sequence of integers $1 = a_0 < a_1 \cdots < a_r = n$, is called an addition chain for n, if $a_i = a_j + a_k$ for $1 \leqq i \leqq r$; $0 \leqq j$, $k < i$. For a given n, the least r for which such a chain exists is called $l(n)$.

Scholz [3] conjectured:

$$l(2^q - 1) \leqq 1(q) + q - 1, \qquad q \geqq 1. \tag{1}$$

A. Brauer [1] proved (1), provided there is a minimal chain $\{a_i\}_{i=1}^{l(q)}$ for q such that $a_i = a_{i-1} + a_t$, $0 < i \leqq l(q)$, $0 \leqq t \leqq i-1$. Gioia, Subbarao, and Sugunamma [2] employ eight lemmas to prove (1) if:

$$q = 2^{c_1} + 2^{c_2} + 2^{c_3} \qquad c_1 > c_2 > c_3 \geqq 0. \tag{2}$$

Lemma 4 of [2] states that, if (2) holds, $l(q) = c_1 + 2$.

It is observed here that (1), subject to (2) follows immediately from this lemma and Brauer's result, since

$$1, 2, 4, \cdots, 2^{c_1}, 2^{c_1} + 2^{c_2}, 2^{c_1} + 2^{c_2} + 2^{c_3}$$

is a minimal chain for q which satisfies Brauer's condition.

REFERENCES

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3. A. Scholtz, *Aufgabe* 253, Jber. Deutsch. Math.-Verein. **47** (1937), 41.

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