$$\alpha = [(\alpha + 1)/2]^2 + i^2[(\alpha - 1)/2]^2.$$

2. If $a = 2a'$ with $a'$ odd and $b$ is even set $\alpha' = a' + ib$. Then

$$\alpha = [(\alpha' + i)/(1 + i)]^2 + i^2[(\alpha' - i)/(1 + i)]^2.$$

3. If $a = 4a'$ and $b$ is odd set $\alpha' = b - 2ia'$. Then

$$\alpha = [(\alpha' + 1)/(1 - i)]^2 + i^2[(\alpha' - 1)/(1 - i)]^2.$$

4. If $a = 4a'$ and $b = 2b'$ set $\alpha' = 2a' + 2ib'$. Then

$$\alpha = [(\alpha' + 2)/2]^2 + i^2[(\alpha' - 2)/2]^2.$$

## References

1. L. J. Mordell, *On the representation of a binary quadratic form as a sum of squares of linear forms*, Math. Z. **35** (1932), 1–15.

2. I. Niven, *Integers of quadratic fields as sums of squares*, Trans. Amer. Math. Soc. **48** (1940), 405–417.

University of Illinois

---

# A MATRIX VERSION OF RENNIE'S GENERALIZATION OF KANTOROVICH'S INEQUALITY

B. MOND

Let $A$ be a positive definite hermetian matrix with eigenvalues $\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_n$. $(A - \lambda_n I)(A - \lambda_1 I)A^{-1}$, where $I$ is the unit matrix, is easily seen to be negative semi-definite since the first factor is positive semi-definite, the second negative semi-definite, the third positive definite and all three commute. Hence, for any $n$-dimensional vector $x$ of unit norm

$$(Ax, x) + \lambda_1\lambda_n(A^{-1}x, x) \leqq \lambda_1 + \lambda_n.$$

Denoting the second term on the left-hand side by $u$,

$$u(Ax, x) \leqq (\lambda_1 + \lambda_n)u - u^2 \leqq (\lambda_1 + \lambda_n)^2/4$$

which is the inequality of Kantorovich. (Cf. B. C. Rennie, *An inequality which includes that of Kantorovich*, Amer. Math. Monthly **70** (1963), 982.)

Aerospace Research Laboratories,
Wright-Patterson Air Force Base, Ohio

Received by the editors March 10, 1965.