5. ———, *Quotient rings of rings with zero singular ideal*, Pacific J. Math. **11** (1961), 1385–1392.

6. ———, *Rings with zero right and left singular ideals*, Trans. Amer. Math. Soc. **118** (1965), 150–157.

7. ———, *Potent rings*, Trans. Amer. Math. Soc. **119**(1965), 524–534.

8. N. Jacobson, *Structure of rings*, Amer. Math. Soc. Colloq. Publ. Vol. **37**, Amer. Math. Soc., Providence, R. I., 1956.

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POWERS IN EIGHTH-GROUPS

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1. Introduction. The purpose of this paper is to give an algorithm which decides whether or not an element in an eighth-group is a power. A group G is an eighth-group if it is finitely presented in the form

$$G = \mathrm{gp}(a_1, \cdots, a_n; R_1(a_\lambda) = 1, \cdots, R_m(a_\lambda) = 1),$$

where (i) each defining relator is cyclically reduced and (ii) if B_i and B_j are cyclic transforms of R_i and R_j, then less than one-eighth of the length of the shorter one cancels in the product $B_i^{\pm 1}B_j^{\pm 1}$, unless the product is unity. The notation in this paper is the same as that in [**3**]. Note that Lemma 3 and Lemma 4 in [**3**] hold for eighth-groups.

Reinhart [**4**] gives an algorithm to decide, among other things, whether or not elements in certain Fuchsian groups are powers. Note that the Fuchsian group $F(p; n_1, \cdots, n_d; m)$, see Greenberg [**1**], is an eighth-group if

$$4p + d + m, n_1, \cdots, n_d > 8.$$

Hence our algorithm holds for a somewhat wider class of groups and, furthermore, is purely algebraic.

Remark. Given any word V in a finitely presented group, it is possible to find a cyclically fully reduced word V^* conjugate to V by writing the word V in a circle and then reducing. Such a word V^* will be called a *reduced cyclic transform* of V.

2. The algorithm. First we prove a lemma about eighth-groups G. Here r denotes the length of the largest defining relator in G.

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LEMMA. *Let W be cyclically fully reduced, let W be conjugate to V, and let $l(V)=n$. Then $l(W)\leqq r^2+rn$.*

PROOF OF LEMMA. By Greendlinger's Basic Theorem in [2, p. 643], there exist reduced cyclic transforms W^* and V^* of W and V such that $W^*=T^{-1}V^*T$, where $l(T)<r/8$ and $l(V^*)\leqq l(V)$. Hence

$$l(T^{-1}V^*T) < r/8 + n + r/8 < r + n.$$

Consequently, by Lemma 3 in [3],

$$l(W^*) \leqq rl(T^{-1}V^*T) \leqq r^2 + rn.$$

But W cyclically fully reduced implies $l(W)=l(W^*)$. Hence the lemma is true.

Suppose, now, that an arbitrary word $W\neq 1$ in an eighth-group is a power, say $W=V^m$ and $l(W)=n$. Let A be a reduced cyclic transform of V; then W is conjugate to A^m. Lemma 4 in [3] implies that $A^m=B$, where B is cyclically fully reduced and where (i) $l(B)\geqq m$, and (ii) $l(B)\geqq l(A)-r$. Accordingly, our lemma above implies

$$m \leqq l(B) < r^2 + nr, \tag{1}$$

$$l(A) \leqq l(B) + r < r^2 + nr + r. \tag{2}$$

The above discussion proves the following

THEOREM. *Let $W\neq 1$ be an arbitrary word in an eighth-group G where $l(W)=n$ and r is the length of the largest defining relator in G. Then W is a power if and only if W is conjugate to A^m where m and A satisfy* (1) *and* (2).

Since the conjugacy problem has been solved for eighth-groups by Greendlinger in [2], and since there exist only a finite number of words in any given length, the above theorem gives us our algorithm.

BIBLIOGRAPHY

1. L. Greenberg, *Discrete groups of motions*, Canad. J. Math. 12 (1960), 415–426.

2. M. Greendlinger, *On Dehn's algorithm for the conjugacy and word problems, with applications*, Comm. Pure Appl. Math. 13 (1960), 641–677.

3. S. Lipschutz, *An extension of Greendlinger's results on the word problem*, Proc. Amer. Math. Soc. 15 (1964), 37–43.

4. B. Reinhart, *Algorithms for Jordan curves on compact surfaces*, Ann. of Math. (2) 75 (1962), 209–222.

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