7. Y. C. Wong, *Existence of linear connections with respect to which given tensor fields are parallel or recurrent*, Nagoya Math. J. 24 (1964), 67–108.

NORTHWESTERN UNIVERSITY

A NOTE ON A REDUCIBLE CONTINUUM

E. L. BETHEL

In [4], Knaster shows that there exists an irreducible compact metric continuum M which has a monotone continuous decomposition G such that each element of G is nondegenerate and M/G is an arc. Also, he raised the question as to whether there existed an irreducible continuum M which has a monotone continuous decomposition G such that each element of G is an arc and M/G is an arc. E. E. Moise settled this question in the negative in [5]. In [3], M. E. Hamstrom showed that if G is a monotone continuous decomposition of a compact metric continuum such that each element of G is a nondegenerate continuous curve and M/G is an arc, then it is not the case that M is irreducible. E. Dyer generalized this result by showing in [2] that if M is a compact metric continuum and G is a monotone continuous decomposition of M such that each element of G is nondegenerate and decomposable, then it is not the case that M is irreducible. A purpose of this note is to extend Dyer's result somewhat.

The author is indebted to the referee for some suggestions which have been incorporated in this note. In particular, a weakened hypothesis in Theorem 2.

THEOREM 1. *Let M denote a compact metric continuum and G a nondegenerate monotone continuous decomposition of M each of whose elements is nondegenerate. If H is a subcollection of G each of whose elements is snakelike and indecomposable, and if H^* is dense in M, then uncountably many elements of G are indecomposable.*

PROOF. Let I_1 denote an element of H, and let C_1 denote the first chain in a sequence of defining chains for I_1, and let L_1 and L_2 denote the end links of C_1. Since H^* is dense in M, and G is a continuous collection, C_1 contains two elements $I(10)$ and $I(11)$ of H such that $I(10)$ and $I(11)$ intersects every link of C_1. Let $\{C_n(10)\}$ and $\{C_n(11)\}$ denote chain sequences which define $I(10)$ and $I(11)$ respectively.

Presented to the Society, November 13, 1965 under the title *On reducibility of continua*; received by the editors January 11, 1965.

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

It follows that there is some $C_i(10)$ of $\{C_n(10)\}$ and some $C_j(11)$ of $\{C_n(11)\}$ such that (1) each is a refinement of C_1, (2) each has links intersecting the first and last links of C_1, and (3) there exist points A_1 and A_2 of L_1 and L_2 respectively, such that if U is a coherent two region collection of either $C_i(10)$ or $C_j(11)$ which intersects A_i, $(i=1, 2)$, then $U^* \subset L_i$, $(i=1, 2)$. Furthermore, the closure of the union of both chains does not intersect I_1, and the diameter of each link is less than 1/2. By Theorem 2 of [1], there is some $C(10)$ of $\{C_n(10)\}$ and a $C(11)$ of $\{C_n(11)\}$ such that $C(10)$ and $C(11)$ each is a refinement of and loop in $C_i(10)$ and $C_j(11)$ respectively. It is an easy exercise to show that both $C(10)$ and $C(11)$ loop in C_1 also.

Now repeat the process used for the construction of $C(10)$ and $C(11)$ in both $C(10)$ and $C(11)$, where $C(10)$ and $C(11)$ assume the role of C_1 and each of $I(10)$ and $I(11)$ assume the role of I_1. By induction, we may define a sequence of chains $\{C(i_1 \cdots i_n)\}$, $(i_k=0, 1)$, such that (1) $I(i_1 \cdots i_n)$ is an element of H which is a subset of the union of the links of $C(i_1 \cdots i_n)$ and intersects each link of $C(i_1 \cdots i_n)$, (2) $C(i_1 \cdots i_n k)$, $(k=0, 1)$, loops and is a refinement of $C(i_1 \cdots i_n)$, and (3) each link of $C(i_1 \cdots i_n)$ has diameter less than $1/n$. Thus, each sequence $\{i_n\}$, $(i_n=0, 1)$, defines a sequence of chains such that the common part of the sequence of chains is an indecomposable continuum I by Theorem 2 of [1]. Since $I(i_1 \cdots i_n)$ intersects each link of $C(i_1 \cdots i_n)$, we have a sequence of elements of G converging to I, and $I \cap I(i_1 \cdots i_n) = \emptyset$, it follows that $I \in G$. Since there are uncountably many sequences $\{i_n\}$, G contains uncountably many snakelike indecomposable continua.

THEOREM 2. *Let M denote a compact irreducible continuum, and let G be a nondegenerate monotone continuous decomposition of M each of whose elements is nondegenerate and either snakelike or decomposable. If M/G has a dense set of separating points, then uncountably many elements of G are indecomposable.*

PROOF. Suppose the contrary. Let G' denote the elements of G which are indecomposable and suppose G' is countable. Now $(G')^*$ is not dense in M since this would imply that G' is uncountable by Theorem 1. Let A and B denote two points between which M is irreducible, and let g denote a separating element of G which does not belong to G'. There exists an open set D with respect to M/G containing g such that $\overline{D} \cap (H \cup g_A \cup g_B) = \emptyset$, where g_A and g_B are the elements of G containing A and B respectively. There is some subcontinuum K of M/G such that K is irreducible from $M/G - D$ to g. It follows that each element of K is decomposable. Since the set of

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use

separating points are dense in M/G, there is a separating point g' of K distinct from g. Furthermore, there is a subcontinuum K' of K irreducible from g' to g. But since each element of K' is decomposable, by Dyer's theorem, there is a proper subcontinuum L of $(K')^*$ intersecting g and g'. Since g and g' are separating points of M/G, it now easily follows that M is not irreducible from A to B, a contradiction. Hence, uncountably many elements of G are indecomposable.

REMARK. May the stipulation that M/G has a dense set of separating points be removed or replaced by a weaker stipulation? Indeed, may the stipulation that the indecomposable elements be snakelike be removed?

REFERENCES

1. L. K. Barrett, *The structure of decomposable snakelike continua*, Duke Math. J. **28** (1961), 515–521.

2. E. Dyer, *Irreducibility of the sum of the elements of a continuous collection of continua*, Duke Math. J. **20** (1953), 589.

3. M. E. Hamstrom, *Continuous collections of continuous curves*, Bull. Amer. Math. Soc. **58** (1952), 204.

4. B. Knaster, *Un continua irreductible à décomposition continue en tranches*, Fund. Math. **25** (1953), 568–577.

5. E. E. Moise, *Monotone interior transformations*, Bull. Amer. Math. Soc. **55** (1949), 810–811.

CLEMSON UNIVERSITY

License or copyright restrictions may apply to redistribution; see https://www.ams.org/journal-terms-of-use