

ON THE GEOMETRY OF STREAMLINES IN HYDRO-MAGNETIC FLUID FLOWS WHEN THE MAGNETIC FIELD IS ALONG A FIXED DIRECTION

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1. **Introduction.** Under the above title Suryanarayan [2] published a paper in which he discusses three-dimensional steady flows of inviscid magnetic fluids. The assumption is that the magnetic field is along a fixed direction and the main goal is to investigate various dynamical and kinematical relations connecting the characteristic properties of the flow and the field quantities with the geometrical parameters of the streamlines. Suryanarayan derives expressions for the tangent, principal normal and binormal vectors, the curvature and torsion of the streamlines in terms of the velocity components, of the pressure, density of the medium and of the intensity of the magnetic field. He obtains the variations of the hydromagnetic pressure along streamlines, along principal normals to the streamlines and shows that this pressure remains constant along the binormals. A special attention is paid to the Bernoulli function; if it exists then the Bernoulli surfaces contain both the streamlines and vortex lines. Suryanarayan finds that the Bernoulli surfaces exist in the case of incompressible fluid and they form the surfaces on which the sum of the fluid pressure, the kinetic energy and the magnetic energy is constant. For an isentropic flow, the variation of the fluid pressure along the streamline is expressed in terms of the Mach number, the magnetic field intensity, the fluid density and the magnitude of the fluid velocity. As a particular case Suryanarayan obtains a class of helical flows. But in his work Suryanarayan has made a tacit assumption that the medium in question possesses the electrical resistivity equal to zero or the electrical conductivity equal to infinity. On one side this assumption results in a very idealistic case, on the other it results in a significant simplification of the equations of continuity, motion and of Maxwell's equations of fields. In the present note the author shows that this tacit assumption is superfluous and the technique of Suryanarayan is applicable to a more general case with a finite magnitude of the electrical resistivity. Below, the author follows precisely the paper of Suryanarayan [2] and gives only equations which differ from those in [2]. The reader is asked to have the work [2] in front of himself during the reading of the present note.

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The left-hand numbers of equations refer to the numbers in Suryanarayan's paper, denoted by the symbol [S].

2. **The equations.** The first equation of Maxwell has the form ($\bar{\mu}$ denotes resistivity) [1]:

$$(S.2.3) \quad u^i \partial_j H_i - H^i \partial_j u_i + H_i \partial_j u^i = \bar{\mu} \partial_j (\partial_j H_i), \tag{2.1}$$

with all the other equations in [S] remaining in their original forms.

3. **The basic decomposition.** Equations (S.3.6), (S.3.11), (S.3.12), (S.3.13) take the forms:

$$(S.3.6) \quad q(dH/ds)h_i - H(dq/dh)s_i - Hq(ds_i/dh) + h_i H [(dq/ds) + q \partial_j s^j] = \bar{\mu} \partial_j (\partial_j H) h_i, \tag{3.1}$$

$$(S.3.11) \quad h_i s^i [d(Hq)/ds + qH \partial_j s^j - \bar{\mu} \partial_j (\partial_j H)] - H(dq/dh) = 0, \tag{3.2}$$

$$(S.3.12) \quad h_i n^i [d(Hq)/ds + qH \partial_j s^j - \bar{\mu} \partial_j (\partial_j H)] - Hq n^i (ds_i/dh) = 0, \tag{3.3}$$

$$(S.3.13) \quad h_i b^i [d(Hq)/ds + qH \partial_j s^j - \bar{\mu} \partial_j (\partial_j H)] - Hq b^i (ds_i/dh) = 0, \tag{3.4}$$

which contain the additional terms containing the factor $\bar{\mu}$ and represent the intrinsic formulation of the fundamental system. Attention is called to the following misprints: (i) in (S.3.4) should be $-\frac{1}{2} \eta h_i h^i \partial_j H^2 = 0$; (ii) in (S.3.17) in the last term on the right-hand side there should be $\dots + u_j (d/ds) (\partial_k P) \}$].

4. **An intrinsic expression for the Bernoulli function.** Here are no changes, except possibly for a few misprints:

$$(S.4.1) \quad \dots = -\partial_i P - \frac{1}{2} \rho \partial_i q^2 + \frac{1}{2} \partial_i (\eta H^2).$$

$$(S.4.4) \quad \dots = \dots + \frac{1}{2} q^2 \rho^{-1} \partial_i \rho = \dots + \frac{1}{2} q^2 \rho^{-1} \partial_i \rho.$$

$$(S.4.8) \quad \partial_i B = - (1/\rho) \left[\partial_i E - \frac{1}{2} q^2 \left(1 - \frac{2}{M^2} \right) \partial_i \rho \right].$$

$$2(dE/ds) = q^2 \left(1 - \frac{2}{M^2} \right) (d\rho/ds).$$

(Kanwal)

$$\frac{1}{\rho q^2} \frac{dp}{ds} = \frac{1}{(M^2 - 1)} \left(\frac{H\eta}{q^2 \rho} \frac{dH}{ds} - \partial_i s^i \right),$$

(notice the change in the sign).

In the lemma below (S.4.8) should be “ . . . if $M = (2)^{1/2}$ and conversely.” In (S.4.6) there is missing the symbol q : . . . = $q[b_k \dots]$.

5. **Helical flows.** As a particular case Suryanarayan [2] calculates the helical flow in the cylindrical coordinates (r, θ, z) . The term containing the resistivity causes the following change in (S.5.4):

$$r^{-1} \sin \beta \frac{\partial}{\partial \theta} (Hq) + \cos \beta \frac{\partial}{\partial z} (Hq) = \bar{\mu} \nabla^2 H, \quad (5.1)$$

$$(S.5.4) \quad \nabla^2 = r^{-1} \partial_r (r \partial_r H) + r^{-2} \partial^2 H / \partial \theta^2, \quad (5.1a)$$

$$\rho q^2 (\sin^2 \beta / r) + \partial P / \partial r = 0, \quad (5.2)$$

with all the other equations remaining as in [S]. This system governs the motion when the streamlines are helices. In the case the variable functions are functions of r only, the governing equations become:

$$dP/dr + \rho q^2 r^{-1} \sin^2 \beta = 0, \quad P = p + \frac{1}{2} \eta H^2, \quad (5.3)$$

$$d^2 H / dr^2 + r^{-1} dH / dr = 0. \quad (5.4)$$

For $\bar{\mu} = 0$, Equation (5.4) does not appear. This is due to the fact that (5.1) vanishes identically. The above conditions give all the helical flows with the flow and field parameters as functions of r . As a special case consider [2], $P = 1 + \exp(-r)$, $q^2 = 2r\rho^{-1}(\exp(-r))$ (see misprints in [2]), $H = A \ln r + B$, $A, B = \text{constants}$, which imply that $\beta = \pi/4$.

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REFERENCES

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2. E. R. Suryanarayan, *On the geometry of streamlines in hydromagnetic fluid flows when the magnetic field is along a fixed direction*, Proc. Amer. Math. Soc. **16** (1965), 90–96.

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