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SYRACUSE UNIVERSITY

A CONDITION FOR A FINITE GROUP TO BE NILPOTENT

STEPHEN MONTAGUE AND GOMER THOMAS

Let $\mathcal{H}$ be a class of groups such that:

(i) If G is in $\mathcal{H}$, then every homomorphic image of G is in $\mathcal{H}$.

(ii) If G is finite and $G/\phi(G)$ is in $\mathcal{H}$, where $\phi(G)$ is the Frattini subgroup of G, then G is in $\mathcal{H}$.

Examples of such classes are the class of nilpotent groups and the class of supersolvable groups. Others can be found in a paper by Baer [1].

In this note a theorem of P. Hall on nilpotent groups is proved as a corollary to the following:

THEOREM. *If G is a finite group with a subgroup H such that $\phi(H)$ is normal in G and $G/\phi(H)$ is in $\mathcal{H}$, then G is in $\mathcal{H}$.*

LEMMA (HUPPERT). *Let G be a finite group, H be a subgroup of G, and N be a subgroup of H such that N is normal in G and $N \leqq \phi(H)$. Then $N \leqq \phi(G)$.*

PROOF. If not, G would have to have a maximal subgroup U such that $N \nleqq U$. Then $H = G \cap H = NU \cap H = N(U \cap H) = U \cap H$, since $N \leqq \phi(H)$. But this implies $H \leqq U$, contrary to $N \nleqq U$.

PROOF OF THEOREM. An application of the Lemma with $N = \phi(H)$ shows that $\phi(H) \leqq \phi(G)$. Hence $G/\phi(G)$ is in $\mathcal{H}$, and so G is in $\mathcal{H}$.

COROLLARY. *If G is a finite group with a normal subgroup H such that H is nilpotent and G/H' is nilpotent, where H' is the commutator subgroup of H, then G is nilpotent.*

PROOF. Since H is nilpotent, $\phi(H)$ contains H'. Hence $G/\phi(H)$ is

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nilpotent, and the theorem applies, with $\mathcal{K}$ being the class of nilpotent groups.

This last result was originally proved by P. Hall [2]. His proof gives an upper bound for the nilpotency class of G in terms of those of H and G/H', and it does not require finiteness of G, but it is fairly complicated.

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University of Illinois