

SHORTER NOTES

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THE CENTER OF A COMPLETE RELATIVELY COMPLEMENTED LATTICE IS A COMPLETE SUBLATTICE

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1. **Introduction.** The purpose of this note is to prove the assertion of its title. The notion of the *center* of a continuous geometry was introduced by J. von Neumann [7, p. 240] who established the fact that the center of such a lattice is a complete sublattice. It is well known [1, p. 27] that the center of any lattice with 0 and 1 is a Boolean sublattice, but it is not known if the center of a complete lattice must be a complete sublattice. Several authors have, however, established this result for various classes of relatively complemented lattices. Specifically, Kaplansky [5, Theorem 5, p. 558] proved it for complemented modular lattices, Foulis [2, Lemma 3, p. 67] as well as Holland [4, p. 72] for orthomodular lattices, and S. Maeda [6, Theorem 3, p. 158] verified it for a relatively semi-orthocomplemented lattice. Our theorem will include all of the above results as special cases.

2. **Proof of the theorem.** The author wishes to thank the referee for suggesting the following proof. Let L be a complete relatively complemented lattice, and let $\{Z_i: i \in I\}$ be a family of central elements of L . Then $Z = \bigwedge_i Z_i$ generates an ideal which is obviously the kernel of a congruence relation. Therefore by Theorems 8 and 11 of [3], Z is neutral. Since L is complemented, Z is in the center of L .

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ADDENDUM TO SOME QUARTIC DIOPHANTINE EQUATIONS OF GENUS 3

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I am indebted to Mr. K. Kloss of the Bureau of Standards, Washington, D. C., for many numerical instances of Theorem III applied to the equation

$$L^3x^3 + M^3y^3 + N^3z^3 = 0.$$

For example, when $a = 7$, $b = 15$, $c = 23$, we can take

p	q	r	L	M	N
8280	4991	13335	12176	6473	–3881
8280	16583	15855	–20512	5297	–353
11040	3703	14175	18208	10313	–6073

These equations, which have no solutions, cannot be proved impossible by taking congruences mod 16.

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