The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

# A NONUNIQUENESS RESULT FOR AN EULER-POISSON-DARBOUX (EPD) PROBLEM

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This note is concerned with a Cauchy problem for a generalization of the EPD equation $\Delta u = kt^{-1}u_t + u_{tt}$. The problem in $m+1$ space-time variables is

$$(1) \qquad \Delta_2 u - b(t;k)u_t - u_{tt} = F(x,t) \qquad (t > 0,\ k \text{ real}),$$

$$(2) \qquad u(x,0) = f(x), \qquad u_t(x,0) = 0;$$

here $b(t;k) = kt^{-1} + B(t)$, and $\Delta_2$ is a Laplace-Beltrami space-operator. It can be assumed that $\Delta_2$ and $f$ are of class $C''$, and that $B$ and $F$ are continuous. Solutions are sought, e.g., that are twice-differentiable above the initial plane $t=0$, and continuously differentiable on the initial plane.

There are several uniqueness results for similar problems, especially for positive index, $k>0$. J. Lions [1] has a uniqueness result for solutions even in $t$ for all $k$ other than the "exceptional" values $-1, -3, -5, \cdots$, under $C^\infty$ conditions. A nonuniqueness result for negative index follows.

*The above Cauchy problem does not have a unique solution for negative index value* $k<0$. This result follows upon noting that the $x$-free function

$$(3) \qquad w = w(t) = \int_0^t \left(\exp \int_\tau^0 B(\sigma)d\sigma\right)\tau^{-k}d\tau \qquad (k < 0)$$

is a solution of the completely homogeneous problem

$$w'' + b(t;k)w' = 0, \qquad w(0) = 0, \qquad w'(0) = 0.$$

This nonuniqueness property persists even if problem (1, 2) is assumed to be analytic.

Choosing $B=0$ reduces (3) to $w=t^{1-k}$, a function first used by A. Weinstein [2] to establish a nonuniqueness result for his EPD equation $\Delta u = kt^{-1}u_t + u_{tt}$, with initial conditions (2) and for negative index $k$.

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## References

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2. A. Weinstein, *On the wave equation and the equation of Euler-Poisson*, Proc. 5th Sympos. Appl. Math., 1952, McGraw-Hill, New York, 1954, pp. 137–147.


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# LOCAL FLATNESS OF COMBINATORIAL MANIFOLDS IN CODIMENSION ONE[1]

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We derive here a fundamental theorem of Brown [1] from a theorem of Cairns [2].

THEOREM. *If $K$ is a combinatorial $n$-manifold without boundary rectilinearly embedded in $R^{n+1}$ then $K$ is locally flat in $R^{n+1}$.*

PROOF. Let $x$ be any point of $K$ and let $v$ be a vertex of $K$ containing $x$ in the interior of its star, $\mathrm{St}(v, K)$. Without loss of generality, we may assume that $v$ is the origin in $R^{n+1}$. The radial projection $\Gamma$ of the link, $\mathrm{Lk}(v, K)$, of $v$ in $K$ on $S^n$ is a combinatorial $(n-1)$-sphere in $S^n$ whose cells are geodesic simplexes on $S^n$. By the main theorem of [2], there is a homeomorphism $h$ of $S^n$ (onto itself) taking $\Gamma(\mathrm{Lk}(v, K))$ onto $S^{n-1}$. Let $h^*$ denote the radial extension of $h$ to a homeomorphism of $R^{n+1}$. Then $h^*$ maps $\mathrm{St}(v, K)$ into $R^n$. Thus $K$ is locally flat in $R^{n+1}$.


## Bibliography

1. M. Brown, *Locally flat embeddings of topological manifolds*, Ann. of Math. **75** (1962), 331–341.

2. S. S. Cairns, *The Schoenflies theorem for polyhedra*, Proc. Nat. Acad. Sci. U.S.A. **47** (1961), 328–330.


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