SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

A NONUNIQUENESS RESULT FOR AN EULER-POISSON-DARBOUX (EPD) PROBLEM

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This note is concerned with a Cauchy problem for a generalization of the EPD equation \( \Delta u = kt^{-1}u_t + u_{tt} \). The problem in \( m+1 \) space-time variables is

\[
\begin{align*}
(1) \quad & \Delta_2 u - b(t; k) u_t - u_{tt} = F(x, t) \quad (t > 0, k \text{ real}), \\
(2) \quad & u(x, 0) = f(x), \quad u_t(x, 0) = 0;
\end{align*}
\]

here \( b(t; k) = kt^{-1} + B(t) \), and \( \Delta_2 \) is a Laplace-Beltrami space-operator. It can be assumed that \( \Delta_2 \) and \( f \) are of class \( C'' \), and that \( B \) and \( F \) are continuous. Solutions are sought, e.g., that are twice-differentiable above the initial plane \( t = 0 \), and continuously differentiable on the initial plane.

There are several uniqueness results for similar problems, especially for positive index, \( k > 0 \). J. Lions [1] has a uniqueness result for solutions even in \( t \) for all \( k \) other than the "exceptional" values \( -1, -3, -5, \cdots \), under \( C^\infty \) conditions. A nonuniqueness result for negative index follows.

The above Cauchy problem does not have a unique solution for negative index value \( k < 0 \). This result follows upon noting that the \( x \)-free function

\[
(3) \quad w = w(t) = \int_0^t \left( \exp \int_0^\tau B(\sigma) d\sigma \right) \tau^{-k} d\tau \quad (k < 0)
\]

is a solution of the completely homogeneous problem

\[
\begin{align*}
\quad & w'' + b(t; k) w' = 0, \quad w(0) = 0, \quad w'(0) = 0.
\end{align*}
\]

This nonuniqueness property persists even if problem (1, 2) is assumed to be analytic.

Choosing \( B = 0 \) reduces (3) to \( w = t^{-k} \), a function first used by A. Weinstein [2] to establish a nonuniqueness result for his EPD equation \( \Delta u = kt^{-1}u_t + u_{tt} \), with initial conditions (2) and for negative index \( k \).

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LOCAL FLATNESS OF COMBINATORIAL MANIFOLDS IN CODIMENSION ONE\textsuperscript{1}

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We derive here a fundamental theorem of Brown [1] from a theorem of Cairns [2].

**Theorem.** If $K$ is a combinatorial $n$-manifold without boundary rectilinearly embedded in $R^{n+1}$ then $K$ is locally flat in $R^{n+1}$.

**Proof.** Let $x$ be any point of $K$ and let $v$ be a vertex of $K$ containing $x$ in the interior of its star, $St(v, K)$. Without loss of generality, we may assume that $v$ is the origin in $R^{n+1}$. The radial projection $\Gamma$ of the link, $Lk(v, K)$, of $v$ in $K$ on $S^n$ is a combinatorial $(n - 1)$-sphere in $S^n$ whose cells are geodesic simplexes on $S^n$. By the main theorem of [2], there is a homeomorphism $h$ of $S^n$ (onto itself) taking $\Gamma(Lk(v, K))$ onto $S^{n-1}$. Let $h^*$ denote the radial extension of $h$ to a homeomorphism of $R^{n+1}$. Then $h^*$ maps $St(v, K)$ into $R^n$. Thus $K$ is locally flat in $R^{n+1}$.

**Bibliography**


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