The result now follows by an easy induction and observing that spheres are s-parallelizable; (the normal bundle of the usual inclusion $S^m \subseteq R^{m+1}$ is ξ and its Whitney sum with τS^m is $\tau R^{m+1} | S^m = \xi^{m+1}$) and $S^{2m+1} \subseteq R^{2m+2} = C^{m+1}$ has a nonzero vector field obtained by multiplying the position vector by i.

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ON CONVEX COMBINATIONS OF BLASCHKE PRODUCTS

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1. If f is analytic in the open unit disk and has modulus bounded by 1, then there is a sequence of finite Blaschke products, $\{E_n\}$, so that $E_n(z) \to f(z)$ for all $|z| < 1$ (see Caratheodory [**1**, p. 13]). The question has been raised by Phelps whether, if f is also assumed continuous on the closed disk, f can be uniformly approximated by convex combinations of finite Blaschke products (see Phelps [**5**] for the motivation of this problem). We do not solve this problem but show the approximation is possible in H_1 norm. We wish to point out that although the work of Nishiura and Waterman [**4**] leaves open the possibility of a Banach-Saks theorem in L_1, it does show that such a theorem is false in H_1.

2. First we claim if f is a sup norm 1 member of the disk algebra, we may approximate f by a polynomial which has no zeros on the boundary of the disk. This polynomial will factor into a finite Blaschke product and a nonvanishing function h. It is clear that to approximate f by a convex combination of Blaschke products it is sufficient to so approximate h. Now h has an analytic square root g with a sup norm bound 1. We apply the Caratheodory result to obtain a sequence of finite Blaschke products $\{E_n\}$ with $E_n(z) \to g(z)$ for $|z| < 1$. Now all the E_n are in H_2 and have H_2 norm 1. The compactness of the H_2 unit ball allows the extraction of a subsequence (for which we use the

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same notation) which converges (the weak topology of H_2 is metric on the unit ball). It is also clear that the weak limit of $\{E_n\}$ is the H_2 function g. Now the Banach-Saks theorem [2, p. 462] will apply and we obtain a further subsequence (and still retain the same notation) which has $C(1)$ means that converge to g in H_2 norm. Then

$$1/K^2 \int |E_1 + \cdots + E_k|^2 d\lambda \to \int |g|^2 d\lambda.$$

and $\|1/N^2 \sum_{n,m}^N E_n E_m\|_1 \to \|h\|_1$. We now need only quote Newman's pseudo uniform convexity for H_1 [3]. If $h_m(z) \to h(z)$ for $|z| < 1$ and $\|h_m\|_1 \to \|h\|_1$ then convergence is in norm.

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