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UNIVERSITY OF NEBRASKA AND
UNIVERSITY OF SOUTHERN CALIFORNIA

# ON COHOMOLOGICAL TRIVIALITY

H. ONISHI

Let $G$ be a finite group and $A$ be a $G$-module. It is well known that if for some two consecutive dimensions the cohomology groups $H^r(H, A)$ are trivial for all subgroups $H$ of $G$, then $H^r(H, A)$ are trivial for all dimensions $r$ and for all subgroups $H$ of $G$. This note is to point out the following generalization.

THEOREM. *Let $G$ be a finite group and $A$ be a $G$-module. If for some integer $k$ and for some odd positive integer $d$, $H^k(H, A)$ and $H^{k+d}(H, A)$ are trivial for all subgroups $H$ of $G$, then $H^r(H, A)$ are trivial for all integers $r$ and for all subgroups $H$ of $G$.*

PROOF. As usual we proceed by induction on the order $n = |G|$. The theorem is trivial for $n=1$. Suppose $n>1$ and assume the truth of the theorem for all groups of order $<n$. In particular, we may assume that $H^r(H, A)=0$ for every dimension $r$ and for every proper subgroup $H$ of $G$. If $n$ is not a prime power, then every Sylow subgroup of $G$ is a proper subgroup and the conclusion follows by a well-known result.

Suppose now $n$ is a prime power, so that $G$ is solvable. Let $H$ be a proper normal subgroup of $G$ such that the quotient group $G/H$ is cyclic. Since $H^r(H, A)=0$ for every $r$, we have the Fundamental Exact Sequence of Hochschild and Serre [2];

$$0 \to H^r(G/H, A^H) \to H^r(G, A) \to H^r(H, A)$$

for every $r>0$. Since the last term is trivial, this says that $H^r(G/H, A^H) \cong H^r(G, A)$ for every $r>0$. By dimension shifting we could have assumed that $k>0$. Thus we have that $H^k(G/A, A^H) = H^{k+d}(G/H, A^H)=0$. Since $G/H$ is cyclic and $d$ is odd, we obtain

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that $H^r(G/H, A^H)=0$ for every $r$. But then by the isomorphism $H^r(G, A)=0$ for every $r>0$. Once we have this, then we obtain that $H^r(G, A)=0$ for every $r\leqq 0$ as well. Q.E.D.

The theorem suggests the following conjecture.

CONJECTURE. Let $G$ be a finite group. Let $A$ and $B$ be $G$-modules and let $f$: $A\rightarrow B$ be a $G$-homomorphism. If $f$ induces isomorphisms

$$H^r(H, A) \cong H^r(H, B)$$

for some two dimensions that differ by an odd integer and for all subgroups $H$ of $G$, then $f$ induces isomorphisms for all dimensions $r$ and for all subgroups $H$ of $G$.

The theorem proved is the special case of the conjecture when $B$ is trivial. L. Evens [1] proved the conjecture when the difference of the dimensions is 1. The conjecture is easily seen to be true when $G$ has a cohomological period by reducing it to the case when $G$ is a $p$-group.

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CITY COLLEGE OF NEW YORK