A REMARK ON THE BROWN-CASLER MAPPING THEOREM

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The Brown-Casler Mapping Theorem [1] states that an arbitrary closed topological manifold M^n can be realized as the image of a map $\phi: I^n \to M$ such that $\phi | \mathring{I}^n$ is a homeomorphism, $\phi^{-1}\phi(\dot{I}^n) = \dot{I}^n$, and $\dim \phi(\dot{I}^n) \leqq n-1$. Of course, each such map gives a *standard decomposition* of M (in the sense of Doyle and Hocking [2]) into an open n-cell and a residual set. Let us call a subset $R \subset M$ *strongly residual* if it can be realized as $\phi(\dot{I}^n)$ for some map ϕ satisfying the Brown-Casler criteria.

PROPOSITION. *Suppose R_1 and R_2 are strongly residual in the same manifold M^n. Then R_1 and R_2 are of the same homotopy type.*

PROOF. Let ϕ_i be the Brown-Casler map corresponding to R_i ($i=1, 2$). Remove a point p from $\mathring{I}^n$, and let $\Psi: I^n - \{p\} \to \dot{I}^n$ be a strong deformation retraction of $I^n - \{p\}$ onto $\dot{I}^n$. Then $\phi_i \circ \Psi \circ \phi_i^{-1}$ is a strong deformation retraction of $M^n - \phi_i(p)$ onto R_i. Since $M^n - \phi_1(p)$ is homeomorphic with $M^n - \phi_2(p)$, it follows that $R_1 \simeq R_2$, completing the proof.

A routine calculation now results in the

COROLLARY. *Let A and B be strongly residual in K^k and M^m, respectively. If C is strongly residual in $K^k \times M^m$, then $C \simeq A \times M^m \cup K^k \times B$. In particular, a strongly residual subset of $S^k \times S^m$ is of the same homotopy type as $S^k \vee S^m$.*

The author has thus far been unable to make significant progress toward classification of strongly residual sets, nor toward determining which Doyle-Hocking residual sets are strongly residual. However, it has been pointed out by several of his colleagues that the proof of the above proposition implies: *each strongly residual set is a compact ANR.*

REFERENCES

1. M. Brown, "A mapping theorem for untriangulated manifolds," *Topology of 3-manifolds*, pp. 92–94, M. K. Fort, Jr. (Editor), Prentice-Hall, Englewood Cliffs, N. J., 1962.

2. P. H. Doyle and J. G. Hocking, *A decomposition theorem for n-dimensional manifolds*, Proc. Amer. Math. Soc. **13** (1962), 469–471.

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