A REMARK ON THE BROWN-CASLER
MAPPING THEOREM

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The Brown-Casler Mapping Theorem [1] states that an arbitrary closed topological manifold \( M^n \) can be realized as the image of a map \( \phi: I^n \rightarrow M \) such that \( \phi|\mathring{I}^n \) is a homeomorphism, \( \phi^{-1}\phi(I^n) = \mathring{I}^n \), and \( \dim \phi(I^n) \leq n - 1 \). Of course, each such map gives a standard decomposition of \( M \) (in the sense of Doyle and Hocking [2]) into an open \( n \)-cell and a residual set. Let us call a subset \( R \subseteq M \) strongly residual if it can be realized as \( \phi(I^n) \) for some map \( \phi \) satisfying the Brown-Casler criteria.

**Proposition.** Suppose \( R_1 \) and \( R_2 \) are strongly residual in the same manifold \( M^n \). Then \( R_1 \) and \( R_2 \) are of the same homotopy type.

**Proof.** Let \( \phi_i \) be the Brown-Casler map corresponding to \( R_i \) \((i = 1, 2)\). Remove a point \( p \) from \( I^n \), and let \( \Psi: I^n \setminus \{p\} \rightarrow I^n \) be a strong deformation retraction of \( I^n \setminus \{p\} \) onto \( I^n \). Then \( \phi_i \circ \Psi \circ \phi_i^{-1} \) is a strong deformation retraction of \( M^n \setminus \phi_i(p) \) onto \( R_i \). Since \( M^n \setminus \phi_1(p) \) is homeomorphic with \( M^n \setminus \phi_2(p) \), it follows that \( R_1 \simeq R_2 \), completing the proof.

A routine calculation now results in the

**Corollary.** Let \( A \) and \( B \) be strongly residual in \( K^k \) and \( M^m \), respectively. If \( C \) is strongly residual in \( K^k \times M^m \), then \( C \simeq A \times M^m \cup K^k \times B \). In particular, a strongly residual subset of \( S^k \times S^m \) is of the same homotopy type as \( S^k \setminus S^m \).

The author has thus far been unable to make significant progress toward classification of strongly residual sets, nor toward determining which Doyle-Hocking residual sets are strongly residual. However, it has been pointed out by several of his colleagues that the proof of the above proposition implies: each strongly residual set is a compact ANR.

**References**


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