

## HOMOTOPY TYPES OF ARNS'S

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Mather (Topology 4 (1965), 92-93) showed that there were only a countable number of homotopy types of compact topological manifolds. His proof used Whitehead's celebrated theorem on homotopy types. The purpose of this note is to give a short direct elementary point-set proof of the following generalization of Mather's result:

**THEOREM.** *There are only countably-many homotopy types of compact metric ANR's.*

**PROOF.** Suppose not. Let  $\{A_\beta\}$  be an uncountable collection of different types. We may regard each  $A_\beta$  as a subset of the Hilbert cube  $I^\omega$ . For each  $\beta$  let  $\epsilon_\beta$  be chosen so that  $A_\beta$  is a retract of its  $\epsilon_\beta$ -neighborhood  $U_\beta$  in  $I^\omega$ . Let  $r_\beta: U_\beta \rightarrow A_\beta$  be a retraction map. Since  $\{A_\beta\}$  is uncountable we may assume (by choosing a subcollection) that there is a positive  $\epsilon$  so that  $\epsilon_\beta \geq \epsilon$  for each  $\beta$ .

Note that if  $f$  and  $g$  are any two maps of a space  $X$  into an  $A_\beta$  so that the distance in  $I^\omega$  between  $f(x)$  and  $g(x)$  is less than  $\epsilon$ , for all  $x$  in  $X$ , then  $f$  and  $g$  are homotopic. In fact  $F(x, t) = r_\beta((1-t)f(x) + tg(x))$  is the desired homotopy, where we use the linear structure in  $I^\omega$ . For each  $\beta$  there is a  $\delta_\beta > 0$  so that  $r_\beta$  restricted to the  $\delta_\beta$ -neighborhood of  $A_\beta$  moves no point as far as  $\epsilon/2$ . Again we may assume that there is a positive  $\delta$  so that  $\delta_\beta \geq \delta$  for each  $\beta$ .

The hyperspace of all closed subsets of  $I^\omega$  with the Hausdorff metric is separable (e.g. if  $D$  is any countable dense set in  $I^\omega$ , the collection of finite subsets of  $D$  is countable and dense in the hyperspace). Since every uncountable set, in a space satisfying the Second Axiom of Countability, contains a limit point, we see that for some  $\beta$  and  $\gamma$ ,  $A_\beta$  is in the  $\delta$ -neighborhood of  $A_\gamma$  and vice versa. Then  $r_\gamma|_{A_\beta}$  and  $r_\beta|_{A_\gamma}$  are homotopy inverses of each other, since their compositions in either direction move points a distance less than  $\epsilon$ , hence, are homotopic to the corresponding identity maps. This shows  $A_\beta$  and  $A_\gamma$  have the same homotopy type, a contradiction.

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Received by the editors November 29, 1966.