HOMOTOPY TYPES OF *ARNS*'S

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Mather (Topology **4** (1965), 92–93) showed that there were only a countable number of homotopy types of compact topological manifolds. His proof used Whitehead's celebrated theorem on homotopy types. The purpose of this note is to give a short direct elementary point-set proof of the following generalization of Mather's result:

THEOREM. *There are only countably-many homotopy types of compact metric ANR's.*

PROOF. Suppose not. Let $\{A_\beta\}$ be an uncountable collection of different types. We may regard each A_β as a subset of the Hilbert cube I^ω. For each β let ϵ_β be chosen so that A_β is a retract of its ϵ_β-neighborhood U_β in I^ω. Let r_β: $U_\beta \rightarrow A_\beta$ be a retraction map. Since $\{A_\beta\}$ is uncountable we may assume (by choosing a subcollection) that there is a positive ϵ so that $\epsilon_\beta \geqq \epsilon$ for each β.

Note that if f and g are any two maps of a space X into an A_β so that the distance in I^ω between $f(x)$ and $g(x)$ is less than ϵ, for all x in X, then f and g are homotopic. In fact $F(x, t) = r_\beta((1-t)f(x) + tg(x))$ is the desired homotopy, where we use the linear structure in I^ω. For each β there is a $\delta_\beta > 0$ so that r_β restricted to the δ_β-neighborhood of A_β moves no point as far as $\epsilon/2$. Again we may assume that there is a positive δ so that $\delta_\beta \geqq \delta$ for each β.

The hyperspace of all closed subsets of I^ω with the Hausdorff metric is separable (e.g. if D is any countable dense set in I^ω, the collection of finite subsets of D is countable and dense in the hyperspace). Since every uncountable set, in a space satisfying the Second Axiom of Countability, contains a limit point, we see that for some β and γ, A_β is in the δ-neighborhood of A_γ and vice versa. Then $r_\gamma | A_\beta$ and $r_\beta | A_\gamma$ are homotopy inverses of each other, since their compositions in either direction move points a distance less than ϵ, hence, are homotopic to the corresponding identity maps. This shows A_β and A_γ have the same homotopy type, a contradiction.

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