

$$p = f - w^*(e - v^*pv)w.$$

The elements pv and $(e - v^*pv)w$ implement the equivalences

$$p \sim v^*pv \quad \text{and} \quad f - p \sim e - v^*pv.$$

Now it follows from Lemma 1 that $f \sim e$.

Von Neumann algebras, W^* -algebras, AW^* -algebras and Baer rings are examples of rings with a complete lattice of projections.

The author wishes to express his appreciation to Bernie Russo whose seminar inspired this paper.

UNIVERSITY OF CALIFORNIA, IRVINE

ON THE FOURIER INVERSION THEOREM FOR R^1

IAN RICHARDS¹

The following is an elementary "noncomputational" proof of the Fourier inversion theorem for tempered distributions on R^1 . The proof does not generalize so easily to R^n , but the inversion theorem for R^n can be deduced from that for R^1 .

To get the inversion theorem for tempered distributions it is sufficient, by duality, to have a proof for the space \mathfrak{D} of *test functions* (i.e. functions $\phi \in C^\infty$ such that $\phi^{(m)}(x) = O(|x|^{-N})$ for all m , $N \geq 0$ as $x \rightarrow \pm \infty$). It is also sufficient to consider only the point $x = 0$.

THEOREM. *There exists a universal constant K such that*

$$(1) \int_{-\infty}^{\infty} \hat{\phi}(t) dt = K\phi(0) \text{ for all } \phi \in \mathfrak{D}.$$

The value of K ($K = 2\pi$) must be determined, as usual, by substituting some particular function ϕ . By linearity, (1) is equivalent to the following:

$$(2) \phi(0) = 0 \text{ implies } \int_{-\infty}^{\infty} \hat{\phi}(t) dt = 0.$$

PROOF OF (2). Since $\phi(0) = 0$, $\psi(x) \equiv (\phi(x)/x) \in C^\infty$. By direct computation, since $\phi(x) = x\psi(x)$, $\hat{\phi}(t) = i(d/dt)\hat{\psi}(t)$. Then, since ψ is also a test function (direct verification), $\int_{-\infty}^{\infty} \hat{\phi}(t) dt = i[\hat{\psi}(\infty) - \hat{\psi}(-\infty)] = 0$. Q.E.D.

UNIVERSITY OF MINNESOTA

Received by the editors October 27, 1966.

¹ This work was partially supported by NSF Grant GP 4033.