let $M-N$ be the interior of a small n-cell containing p.) Since X is $(n-2)$-connected, $H_i(X)=0$ for $0<i\leqq n-2$ and $H_0(X)\cong Q$ ([5, p. 349]). In Čech homology theory on the category of compact pairs every triad is a proper triad ([3, p. 266]). Therefore, we may apply the Mayer-Vietoris sequence to the triad $(N, (E\cup X)\cap N, M-E)$ and conclude that $H_i(M-E)=0$, for $0<i\leqq n-2$, and $H_0(M-E)\cong Q$. Since $M-E$ is a proper subset of a connected n-manifold, it follows that $H_i(M-E)=0$ for $i\geqq n$. (We assume here that $n>0$. If $n=0$, the theorem is trivial.) By Alexander duality, (see [6, p. 263]) since E is arc-wise connected, $H_{n-1}(M-E)=0$. Therefore $M-E$ is acyclic and the theorem follows.

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A T_1-COMPLEMENT FOR THE REALS

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The family of all topologies definable on an arbitrary set X forms a complete lattice Σ under the partial ordering: $\tau_1\leqq\tau_2$ if and only if $\tau_1\subseteq\tau_2$. The lattice operations $\wedge$ and $\vee$ are defined as: $\tau_1\wedge\tau_2=\tau_1\cap\tau_2$ and $\tau_1\vee\tau_2$ is the topology generated by the base $\mathcal{B}=\{B: B=U_1\cap U_2, U_1\in\tau_1 \text{ and } U_2\in\tau_2\}$. The greatest element, 1, is the discrete topology and the least element, 0, is the trivial topology. The lattice Σ has been recently studied [2], [3], [4] and has been shown to be complemented [4].

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The family of all T_1-topologies definable on X forms a complete sublattice Λ of Σ, with greatest element 1, and least element, the cofinite topology $\lambda=\{U: U=\emptyset \text{ or } X-U \text{ is finite}\}$. However, an example has been given in [**5**] to show that Λ is not a complemented lattice, unless X is a finite set.

The question as to which T_1-topologies have T_1-complements has been studied in [**1**], [**5**], [**6**]. Although large classes of T_1-topologies have been shown to have T_1-complements, the most common spaces are not included. In fact, the question concerning the real numbers has been outstanding for some time.

We have shown in [**6**] that the space of real numbers with the usual topology has a T_1-complement if any countable dense subspace has a T_1-complement. The purpose of this paper is to use this fact to produce a complement for the reals.

Let (R, μ) be the real numbers R with the usual topology μ, Q be the rational numbers and D be the dyadic rationals. We now will define a countable dense subspace X of R.

For each integer k, let $S_{0,k}$ be a sequence in $(Q-D)\cap\{(k, k+1/2)\}$ which converges to k and let $A_0=\bigcup\{S_{0,k} \mid k=0, \pm 1, \pm 2, \cdots\}$.

For each integer k, let $S_{1,k}$ be a sequence in $(Q-D)\cap\{(k-1/2, k)\}$ which converges to $k-1/2$ and let $A_1=\bigcup\{S_{1,k} \mid k=0, \pm 1, \pm 2, \cdots\}$.

Since D is countable, $D-\{r \mid r=k \text{ or } k+1/2, k \text{ an integer}\}$ can be ordered as $\{d_1, d_2, \cdots\}$. There is a bounded open interval I_1 containing d_1 such that $I_1\cap(A_0\cup A_1)=\emptyset$. Let S_1 be a sequence in $I_1\cap(Q-D)$ converging to d_1. Suppose S_p has been chosen for each $p<n$. There is a bounded open interval I_n containing d_n such that $I_n\cap[(A_0\cup A_1)\cup(\bigcup\{S_p \mid p<n\})]=\emptyset$. Let S_n be a sequence in $I_n\cap(Q-D)$ converging to d_n.

Let $X=D\cup A_0\cup A_1\cup(\bigcup\{S_n \mid n=1, 2, \cdots\})$ and τ be the relative topology on X with respect to μ.

Define a topology τ' on X to be the topology generated by sets of the form:

(i) $\{x\}$, $x\in D$,

(ii) U, $U\in\lambda$ where λ is the cofinite topology on X,

(iii) B_i, $i=0, 1$,

(iv) C_i, $i=1, 2, \cdots$,

where

$$B_0=A_0\cup\{(X-A_1)\cap(\bigcup\{[k-1/2, k) \mid k \text{ an integer}\})\},$$
$$B_1=A_1\cup\{(X-A_0)\cap(\bigcup\{[k, k+1/2) \mid k \text{ an integer}\})\},$$

and

$$C_i=S_i\cup\{(X-I_i)\cap D\}.$$

We notice that $B_0 \cup B_1 = X$ and that $C_i \cap C_j \subset D$ if $i \neq j$. Since $\lambda \subset \tau'$, τ' is a T_1-topology. We will show that $\tau \vee \tau' = 1$ and $\tau \wedge \tau' = \lambda$.

(a) $\tau \vee \tau' = 1$. Let $x \in X$. If $x \in D$ then $\{x\} \in \tau'$. If $x \in S_i$ then there is an open interval U containing x such that $U \subset I_i$ and $U \cap S_i = \{x\}$. Thus $\{x\} = (U \cap X) \cap C_i \in \tau \vee \tau'$. If $x \in A_0$ then there is an open interval U containing x such that $U \subset (k, k+1/2)$ for some k and $U \cap A_0 = \{x\}$. Thus $\{x\} = (U \cap X) \cap B_0 \in \tau \vee \tau'$. A similar argument holds if $x \in A_1$.

(b) $\tau \wedge \tau' = \lambda$. Let $U \in \tau \wedge \tau'$ and suppose $U \neq \varnothing$. Since $U \in \tau$, U must contain elements of D; so let $x \in D \cap U$. If $x = d_n$ for some n, then all but a finite number of elements of S_n must be in U. Thus almost all of $C_n \cap B_0$ or almost all of $C_n \cap B_1$ is contained in U (since these are the only base elements in τ', other than C_n or members of λ, which contain almost all of S_n). But if a cofinite subset of $C_n \cap B_1$ is in U then U contains almost all the integers and hence U must contain a cofinite subset of B_0 since B_0 is the only base element in τ' containing the sequences which converge to the integers. But B_0 contains all dyadic rationals of the form $(2k+1)/2$; so a cofinite subset of B_1 must be contained in U and therefore $U \in \lambda$.

The other cases where a cofinite subset of $C_n \cap B_0$ is contained in U or where $x \in D - \{d_1, d_2, \cdots\}$ are treated in a similar fashion.

Thus τ' is a T_1-complement for τ and since X is a countable dense subset of R, μ also has a T_1-complement. The elements of this complement may be obtained from those in τ' by following the construction given in [**6**].

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