let $M - N$ be the interior of a small n-cell containing p.) Since X is $(n-2)$-connected, $H_i(X) = 0$ for $0 < i \leq n-2$ and $H_0(X) \cong Q$ ([5, p. 349]). In Čech homology theory on the category of compact pairs every triad is a proper triad ([3, p. 266]). Therefore, we may apply the Mayer-Vietoris sequence to the triad $(N, (E \cup X) \cap N, M - E)$ and conclude that $H_i(M - E) = 0$, for $0 < i \leq n-2$, and $H_0(M - E) \cong Q$. Since $M - E$ is a proper subset of a connected n-manifold, it follows that $H_i(M - E) = 0$ for $i \geq n$. (We assume here that $n > 0$. If $n = 0$, the theorem is trivial.) By Alexander duality, (see [6, p. 263]) since E is arc-wise connected, $H_{n-1}(M - E) = 0$. Therefore $M - E$ is acyclic and the theorem follows.

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A T_1-COMPLEMENT FOR THE REALS

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The family of all topologies definable on an arbitrary set X forms a complete lattice Σ under the partial ordering: $\tau_1 \leq \tau_2$ if and only if $\tau_1 \subseteq \tau_2$. The lattice operations $\wedge$ and $\vee$ are defined as: $\tau_1 \wedge \tau_2 = \tau_1 \cap \tau_2$ and $\tau_1 \vee \tau_2$ is the topology generated by the base $\mathfrak{B} = \{B : B = U_1 \cap U_2, U_1 \in \tau_1$ and $U_2 \in \tau_2\}$. The greatest element, 1, is the discrete topology and the least element, 0, is the trivial topology. The lattice Σ has been recently studied [2], [3], [4] and has been shown to be complemented [4].

The family of all T_1-topologies definable on X forms a complete sublattice Λ of Σ, with greatest element 1, and least element, the cofinite topology $\lambda = \{U: U = \varnothing$ or $X - U$ is finite$\}$. However, an example has been given in [5] to show that Λ is not a complemented lattice, unless X is a finite set.

The question as to which T_1-topologies have T_1-complements has been studied in [1], [5], [6]. Although large classes of T_1-topologies have been shown to have T_1-complements, the most common spaces are not included. In fact, the question concerning the real numbers has been outstanding for some time.

We have shown in [6] that the space of real numbers with the usual topology has a T_1-complement if any countable dense subspace has a T_1-complement. The purpose of this paper is to use this fact to produce a complement for the reals.

Let (R, μ) be the real numbers R with the usual topology μ, Q be the rational numbers and D be the dyadic rationals. We now will define a countable dense subspace X of R.

For each integer k, let $S_{0,k}$ be a sequence in $(Q-D) \cap \{(k, k+1/2)\}$ which converges to k and let $A_0 = \cup\{S_{0,k} | k = 0, \pm 1, \pm 2, \cdots\}$.

For each integer k, let $S_{1,k}$ be a sequence in $(Q-D) \cap \{(k-1/2, k)\}$ which converges to $k-1/2$ and let $A_1 = \cup\{S_{1,k} | k = 0, \pm 1, \pm 2, \cdots\}$.

Since D is countable, $D - \{r | r = k$ or $k+1/2$, k an integer$\}$ can be ordered as $\{d_1, d_2, \cdots\}$. There is a bounded open interval I_1 containing d_1 such that $I_1 \cap (A_0 \cup A_1) = \varnothing$. Let S_1 be a sequence in $I_1 \cap (Q-D)$ converging to d_1. Suppose S_p has been chosen for each $p < n$. There is a bounded open interval I_n containing d_n such that $I_n \cap [(A_0 \cup A_1) \cup (\cup\{S_p | p < n\})] = \varnothing$. Let S_n be a sequence in $I_n \cap (Q-D)$ converging to d_n.

Let $X = D \cup A_0 \cup A_1 \cup (\cup\{S_n | n = 1, 2, \cdots\})$ and τ be the relative topology on X with respect to μ.

Define a topology τ' on X to be the topology generated by sets of the form:

(i) $\{x\}$, $x \in D$,
(ii) U, $U \in \lambda$ where λ is the cofinite topology on X,
(iii) B_i, $i = 0, 1$,
(iv) C_i, $i = 1, 2, \cdots$,

where
$$B_0 = A_0 \cup \{(X - A_1) \cap (\cup\{[k-1/2, k) | k \text{ an integer}\})\},$$
$$B_1 = A_1 \cup \{(X - A_0) \cap (\cup\{[k, k+1/2) | k \text{ an integer}\})\},$$
and
$$C_i = S_i \cup \{(X - I_i) \cap D\}.$$

We notice that $B_0 \cup B_1 = X$ and that $C_i \cap C_j \subset D$ if $i \neq j$. Since $\lambda \subset \tau'$, τ' is a T_1-topology. We will show that $\tau \vee \tau' = 1$ and $\tau \wedge \tau' = \lambda$.

(a) $\tau \vee \tau' = 1$. Let $x \in X$. If $x \in D$ then $\{x\} \in \tau'$. If $x \in S_i$ then there is an open interval U containing x such that $U \subset I_i$ and $U \cap S_i = \{x\}$. Thus $\{x\} = (U \cap X) \cap C_i \in \tau \vee \tau'$. If $x \in A_0$ then there is an open interval U containing x such that $U \subset (k, k+1/2)$ for some k and $U \cap A_0 = \{x\}$. Thus $\{x\} = (U \cap X) \cap B_0 \in \tau \vee \tau'$. A similar argument holds if $x \in A_1$.

(b) $\tau \wedge \tau' = \lambda$. Let $U \in \tau \wedge \tau'$ and suppose $U \neq \emptyset$. Since $U \in \tau$, U must contain elements of D; so let $x \in D \cap U$. If $x = d_n$ for some n, then all but a finite number of elements of S_n must be in U. Thus almost all of $C_n \cap B_0$ or almost all of $C_n \cap B_1$ is contained in U (since these are the only base elements in τ', other than C_n or members of λ, which contain almost all of S_n). But if a cofinite subset of $C_n \cap B_1$ is in U then U contains almost all the integers and hence U must contain a cofinite subset of B_0 since B_0 is the only base element in τ' containing the sequences which converge to the integers. But B_0 contains all dyadic rationals of the form $(2k+1)/2$; so a cofinite subset of B_1 must be contained in U and therefore $U \in \lambda$.

The other cases where a cofinite subset of $C_n \cap B_0$ is contained in U or where $x \in D - \{d_1, d_2, \cdots\}$ are treated in a similar fashion.

Thus τ' is a T_1-complement for τ and since X is a countable dense subset of R, μ also has a T_1-complement. The elements of this complement may be obtained from those in τ' by following the construction given in [6].

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