

A SEMISHRINKING BASIS WHICH IS NOT SHRINKING

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A Schauder basis (x_i, f_i) for a Banach space X is

- (1) *shrinking* provided (f_i) is a basis for X^* ; and
- (2) *semishrinking* provided $0 < \inf_n \|x_n\| \leq \sup_n \|x_n\| < +\infty$ and (x_n) is weakly convergent to 0.

A. Pełczyński and W. Szlenk [2], answering a question raised by I. Singer, constructed a Banach space with a semishrinking basis which was not shrinking. Their construction, while elegant, is very complicated. Our purpose here is to show that the space (d) constructed by Davis and Dean [1, p. 214] has such a basis.

The space (d) consists of those real sequences $a = (a_i)$ for which

$$\|a\| = \sup_{p \in \mathcal{O}} \sum_{i=1}^{\infty} \frac{|a_{p(i)}|}{i} < +\infty$$

where \mathcal{O} denotes the collection of all 1-1 maps from the positive integer ω into ω . Davis and Dean [1, p. 214] have shown that the unit vectors (e_n) form a semishrinking basis for (d) (although their terminology is different). We show that (e_n) is not shrinking.

Let $h_n = \sum_{i=1}^n (1/i)$ and define $u_n \in (d)$ by $u_n = h_n^{-1} \sum_{i=1}^n e_i$. Clearly $\|u_n\| = 1$ for $n = 1, 2, 3, \dots$. Define a linear functional F on (d) by $F(x) = \sum_{i=1}^{\infty} (x_i/i)$ where $x = (x_i)$. By the definition of the norm on (d) it follows that $F \in (d)^*$. Moreover, $F(u_n) = 1$ for each n and so (u_n) does not converge weakly to 0. However if (f_i) denotes the coefficient functionals associated with (e_n) then $\lim_{n \rightarrow \infty} f_i(u_n) = \lim_{n \rightarrow \infty} h_n^{-1} = 0$. Thus by a theorem of Wilansky [4] (see also [3, property (E)]) (e_n) is not shrinking.

BIBLIOGRAPHY

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