A REMARK ON REPRESENTATIONS OF CCR ALGEBRAS

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Let A be a CCR algebra, that is, a C^*-algebra whose images under all irreducible representations consist of compact operators. Let $\hat{A}$ be dual of A. We shall show the following

THEOREM. *Let $\pi_1, \pi_2, \cdots, \pi_n$ be distinct points in $\hat{A}$ and $T_1, T_2, \cdots, T_n$ be given compact operators in each representing space of $\pi_1, \pi_2, \cdots, \pi_n$. Then there exists an element x in A such that $\pi_1(x)=T_1$, $\pi_2(x)=T_2, \cdots, \pi_n(x)=T_n$.*

This is an affirmative answer to the problem raised in Dixmier [2, §3, No. 8] in the International Symposium on C^*-algebras at Louisiana State University in March, 1967.

Let P_i be the kernel of the representation π_i, then $P_1, P_2, \cdots, P_n$ are distinct maximal ideals in A from each other. We assume $n>1$ and suppose for some k $(<n)$ there exists an element $y\in A$ such that $\pi_1(y)=T_1$, $\pi_2(y)=T_2, \cdots, \pi_k(y)=T_k$. We shall show that there exists an element $x\in A$ such that $\pi_1(x)=T_1$, $\pi_2(x)=T_2, \cdots, \pi_{k+1}(x)=T_{k+1}$.

At first, we notice that the ideal $P_1\cap P_2\cap \cdots \cap P_k$ can not be contained in P_{k+1} because A is CCR and the set $\{P_1, P_2, \cdots, P_k\}$ is closed in Prim(A), the ideal structure space of A (cf. [1]). Hence, $P_{k+1} \subsetneqq P_1\cap P_2\cap \cdots \cap P_k+P_{k+1}$ and the latter is dense in A. Therefore, by [3, Lemma 8.1],

$$P_1 \cap P_2 \cap \cdots \cap P_k + P_{k+1} = A.$$

Take an element $z\in A$ such as $\pi_{k+1}(z)=T_{k+1}$ and put $y-z=h-k$ where $h\in P_{k+1}$ and $k\in P_1\cap P_2\cap \cdots \cap P_k$. Then it is clear that the element $x=y+k=z+h$ satisfies the condition, $\pi_1(x)=T_1$, $\pi_2(x)=T_2, \cdots, \pi_k(x)=T_k$, $\pi_{k+1}(x)=T_{k+1}$. Thus, by mathematical induction we get the conclusion of the theorem.

REFERENCES

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