sents a real circle with a positive radius having the origin in its interior (because the constant term in this equation is negative); and when c, d are not both zero, the straight line represented by the equation (4) meets this circle in two real and distinct points. We can, therefore, always find (at least) two distinct complex numbers z_k such that $z=z_k$ satisfy the equation (1). This proves slightly more than what we set out to prove.

For real inner product spaces $\mathfrak{X}$ we may similarly reduce the proof of the corresponding theorem to showing that a certain quadratic equation with real coefficients has real roots.

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A NOTE ON ABSOLUTE SUMMABILITY[1]

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Let A be an infinite matrix defining a sequence to sequence mapping by $(Ax)_n = \sum_k a_{nk}x_k$. The purpose of this note is to present a short elementary proof of the result that characterizes l-l methods (if $\sum_k |x_k|$ converges, then $\sum_n |(Ax)_n|$ converges). The proof in [2] is complicated by the fact that A is applied to the sequence of partial sums, rather than to x itself. Although the proof of Knopp and Lorentz [1] is elegant, it depends on the Principle of Uniform Boundedness.

THEOREM. *The matrix A defines an l-l method if and only if there is a number M such that for each k*

$$(*) \qquad \sum_n |a_{nk}| \leqq M.$$

PROOF. The sufficiency of (*) is easy since it yields

$$\sum_n |(Ax)_n| \leqq M \sum_k |x_k|.$$

If A is an l-l method, it is clear that each row sequence of A must be bounded: say $|a_{nk}| \leqq B_n$ for each k. It is also obvious that each

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column sequence must be a member of l. However, suppose that the sequence μ, given by $\mu_k=\sum_n |a_{nk}|$, is unbounded.

Choose $\kappa(1)$ so that $\mu_{\kappa(1)}>2$, and define $\eta(1)$ such that

$$\sum_{n=1}^{\eta(1)} |a_{n,\kappa(1)}| > 2.$$

Having defined $\kappa(i)$ and $\eta(i)$ for each i less than m, let $\kappa(m)$ be greater than $\kappa(m-1)$ and satisfy

$$\sum_{n=1+\eta(m-2)}^{\eta(m-1)} \sum_{k\geq\kappa(m)} B_n k^{-2} < 1$$

and

$$\mu_{\kappa(m)} > m^2\left\{2+\sum_{i<m} i^{-2}\mu_{\kappa(i)}\right\} + \sum_{n\leq\eta(m-1)} B_n.$$

We can now choose $\eta(m)$ such that

$$\sum_{n=1+\eta(m-1)}^{\eta(m)} |a_{n,\kappa(m)}| > m^2\left\{2+\sum_{i<m} i^{-2}\mu_{\kappa(i)}\right\}.$$

It now follows that, if $x_k=0$ when $k\neq\kappa(i)$ and $x_{\kappa(i)}=i^{-2}$, then Ax is not in l. For

$$\begin{aligned}\sum_{n=1+\eta(m-1)}^{\eta(m)} |(Ax)_n| &\geqq \sum_{n=1+\eta(m-1)}^{\eta(m)} |a_{n,\kappa(m)}|\, m^{-2} \\ &\quad - \sum_{n=1+\eta(m-1)}^{\eta(m)} \sum_{i<m} |a_{n,\kappa(i)}|\, i^{-2} \\ &\quad - \sum_{n=1+\eta(m-1)}^{\eta(m)} \sum_{i\geq\kappa(m+1)} B_n i^{-2} \\ &> 1.\end{aligned}$$

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