CORRECTION TO MY PAPER "THE MORSE INDEX THEOREM"

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Professors Louis Nirenberg and William T. Reid have very kindly pointed out lacunae in [1]: (i) one does not verify that the symmetric matrix $(I(v_i, v_j))$ is negative definite by computing only its diagonal terms, and (ii) there is an inconsistency in the two characterizations of the subspace $H^+ \subset H$. Fortunately the method of [1] is easily modified to correct these deficiencies, according to suggestions of Nirenberg and Reid presented below. Further remarks appear in [2].

Let H^+ be the subspace of those $u \in H$ such that $u(t_i)$ lies in the image of $U(t_i)$ for each of the r focal points t_i, as on p. 761 of [1]. This definition of H^+ is broader than that of p. 760 since for $r>0$ one can always find elements in the present H^+ which are images under U of arc-wise C^∞ curves with discontinuities at the focal points, but not images under U of C^∞ curves. However, $U'U^{-1}$ is still a linear map $V: H^+ \to G$, and if $u(t_i) = U(t_i)v(t_i^+) = U(t_i)v(t_i^-)$ with $v(t_i^+) \neq v(t^-{}_i)$ at the focal point t_i, one proceeds as follows, with some obvious notational abbreviations. The self-adjointness of ${}^tUU'$ gives $(U'v^+, Uv^-) = (Uv^+, U'v^-)$ at t_i, hence

$$\begin{aligned}(Vu, u)(t_i^+) - (Vu, u)(t_i^-) &= (U'v^+, Uv^+) - (U'v^-, Uv^-) \\ &= (U'v^+, Uv^+ - Uv^-) + (U'v^+ - U'v^-, Uv^-) \\ &= (U'v^+, Uv^+ - Uv^-) + (Uv^+ - Uv^-, U'v^-) \\ &= 0;\end{aligned}$$

thus the function (Vu, u) is continuous even at the focal points so that the first integral formula is valid for the broader definition of H^+, implying $I(u) \geqq 0$ for $u \in H^+$ as before.

Now let H^- be any maximal subspace on which I is negative definite, so that $H^+ \cap H^- = \{0\}$, and let $q = \dim H^-$; since H^+ is of codimension r it follows that $q \leqq r$ because $H^+ \cap H^-$ would otherwise be nontrivial. Let K be the subspace of those $u \in H$ with $I(u, v) = 0$ for all $v \in H^-$; then K is of codimension q since I is negative definite on H^-, and the relation $K \cap H^- = \{0\}$ implies $K + H^- = H$. Furthermore if $I(v) < 0$ for some $v \in K$ then for any $u \in H^-$ and real number t

Received by the editors October 2, 1967 and, in revised form, February 17, 1968.

[1] Research supported by National Science Foundation Grant NSF-GP-5477.

the relation $I(u, v)=0$ would imply $I(u+tv)=I(u)+t^2I(v)<0$ except when $u=0$ and $t=0$, contrary to the maximality of H^-. Thus I is positive semidefinite on K.

Let L be the r-dimensional subspace spanned by the linearly independent functions $u_1, \cdots, u_r$ of [1]; then I vanishes on L and every nonzero element of L has a discontinuous derivative at at least one focal point. If $q<r$ there is a nonzero $u_0 \in K \cap L$ since K is of codimension q, and the restriction of I to K is minimized by u_0; in fact $I(u_0)=0$ because $u_0 \in L$. This implies $I(u_0, u)=0$ for any $u \in K$, and since $I(u_0, u)=0$ for any $u \in H^-$ it follows that $I(u_0, u)=0$ for any $u \in H$; consequently u_0 is everywhere C^∞ by the Lemma of [1] (with a trivial change of wording), contradicting the earlier remark about nonzero elements of L. The hypothesis $q<r$ is thus at fault, and one is left with the conclusion that H^- has a basis $v_1, \cdots, v_r$. The remainder of the proof of the Morse index theorem can now be completed as in [1].

References

1. H. Osborn, *The Morse index theorem*, Proc. Amer. Math. Soc. **18** (1967), 759–762.

2. W. T. Reid, *Remarks on the Morse index theorem*, Proc. Amer. Math. Soc. **20** (1969), 339–341.

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