A CORRECTION TO "SOME BOUNDARY PROPERTIES OF THE RIEMANN MAPPING FUNCTION"

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The paper [1], dealing with applications of boundary properties of the Riemann mapping function, makes essential use of a rectifiability hypothesis on the boundary. Michael R. Cullen has kindly called the author's attention to the fact that the form in which this hypothesis is stated differs from that actually used in the derivations. However, a simple rephrasing of the underlying conventions on the region Ω suffices to set matters straight, and we present the necessary changes here, along with some related comments.

A generalization of the classical Osgood-Taylor-Carathéodory theorem serves as the starting point in [1]. Let Ω be a bounded simply connected plane region for which $\partial\Omega$ can be parametrized as a closed curve. Then any Riemann mapping function χ for Ω, i.e. any function mapping the open unit disc ω conformally onto Ω, can be extended to a continuous mapping of $\bar{\omega}$ onto $\bar{\Omega}$. In stating the rectifiability hypothesis in [1] a parallel wording was used, namely that $\partial\Omega$ be parametrizable as a rectifiable closed curve. The condition actually employed in the derivations, however, is that of rectifiability of disc-induced parametrizations of the prime-end boundary. While the two conditions are probably equivalent, a proof does not appear to be obvious. On the other hand, the whole question can be regarded as peripheral to the analytical results of [1], since these would normally be applied (as in the case of the disc slit along a radius) by tracing out the prime-end boundary in the natural way.

As discussed in [1], the generalized Osgood-Taylor-Carathéodory theorem ensures that the prime-end boundary can be parametrized as a Jordan curve, the so-called *Jordan-Carathéodory boundary curve* Λ. Since any two Riemann mapping functions χ are connected by a linear fractional transformation, it is clear that different choices of χ yield equivalent parametrizations of Λ. Thus, the notions of Jordan-Carathéodory boundary curve and arc length along such a curve are intrinsic.

In the light of these remarks we revise Theorem 1 of [1] as

THEOREM 1. *Let Ω be a bounded simply connected plane region and* χ

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a Riemann mapping function for Ω. *Then* χ' *is in the Hardy class* H^1 *if and only if*

(1) $\partial\Omega$ *can be parametrized as a closed curve and*

(2) *the Jordan-Carathéodory boundary curve for* Ω *is rectifiable.*

A corresponding revision of the blanket conventions for the remainder of [1] can be stated as follows. *Let* Ω *be given as a bounded plane region, and suppose that* $\partial\Omega$ *consists of finitely many disjoint continua, each parametrizable as a closed curve for which the corresponding Jordan-Carathéodory boundary curve is rectifiable.* As before, Λ will be used to designate the positively oriented Jordan-Carathéodory boundary of Ω, and arc length along Λ will be denoted by s.

Reference

1. M. G. Arsove, *Some boundary properties of the Riemann mapping function*, Proc. Amer. Math. Soc. **19** (1968), 560–567.

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