A COUNTABLE MINIMAL URYSOHN SPACE IS COMPACT

R. M. STEPHENSON, JR.

1. **Introduction.** A topological space X is said to be a Urysohn space provided that every pair of distinct points of X have disjoint closed neighborhoods. In [**3**] C. T. Scarborough and A. H. Stone prove that a countable minimal regular space is compact, and in [**4**] Scarborough asks if there exist noncompact minimal Urysohn spaces which are countable.

The principal result of this note is the following:

THEOREM. *A countable minimal Urysohn space is compact.*

The terminology used here coincides with that in [**3**] and [**4**]. We shall denote the set of positive integers by N.

2. **The theorem.** According to [**4**, Theorem **14**], a countable U-closed space has an isolated point. Our first lemma strengthens this result.

LEMMA 1. *Let X be a U-closed space, let I be its set of isolated points, and suppose that $X-I$ is countable. Then $\bar{I}=X$.*

PROOF. Suppose that there exists a nonempty open set V such that $V\cap I=\emptyset$. Let $X-I=\{x_n|n\in N\}$. An inductive argument shows that there exists a descending sequence V_n, $n\in N$, of nonempty open subsets of V such that for every $k\in N$ there exists a neighborhood W_k of x_k whose closure misses the closure of V_k. Then the filter base $\{V_n|n\in N\}$ is a U-filter on X which has void adherence.

LEMMA 2. *Let X be a U-closed space, I its set of isolated points, and $\mathfrak{F}$ a countable filter base on I. Then $\mathfrak{F}$ has an adherent point.*

PROOF. Let $\mathfrak{F}=\{F_n|n\in N\}$. For each n choose a point $x_n\in\bigcap\{F_i|i\leqq n\}$ and let $C=\{x_n|n\in N\}$. If there is a point $x\in\bar{C}-C$, then x is an adherent point of $\mathfrak{F}$. If $\bar{C}=C$ and $\bigcap\mathfrak{F}=\emptyset$, then the open- and-closed sets $\{x_j|j\geqq n\}$, $n\in N$, generate a U-filter on X which has void adherence.

PROOF OF THE THEOREM. Let X be a countable minimal Urysohn space.

In [**4**] Scarborough proves that a minimal Urysohn space is semiregular. In [**2**] Katětov proves that a semiregular absolutely closed space is minimal Hausdorff and that a Urysohn minimal Hausdorff

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space is compact (this latter result is also obtained in [1]). Thus it suffices for us to prove that X is absolutely closed.

Since X is a Lindelöf space, an easy argument shows that if every countable open filter base on X has an adherent point, then X is absolutely closed.

Let $\mathfrak{F}$ be a countable open filter base on X, and let I be the set of isolated points of X. Then $\mathfrak{F}|I$ is a filter base by Lemma 1, so $\mathfrak{F}|I$ and, hence, $\mathfrak{F}$ have nonvoid adherence by Lemma 2.

COROLLARY. *A countable U-closed space is absolutely closed.*

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UNIVERSITY OF NORTH CAROLINA AT CHAPEL HILL