ON SOME SUBSPACES OF BANACH SPACES
WHOSE DUALS ARE $L_1$ SPACES

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1. Introduction. Banach spaces whose duals are $L_1$ spaces were extensively studied in [3]. Recently J. Lindenstrauss and D. Wulbert [4] proved some results on the classification of these spaces which complemented the results of [3]. In [2] the results of [5] were applied to prove the following structure theorem for the separable case:

**Proposition 1** [2, Theorem 2]. A separable Banach space $X$ satisfies $X^* = L_1$ space if and only if $X$ has a monotone basis $\{x_i\}_{i=1}^\infty$ such that for every $n$ the span of $\{x_i\}_{i=1}^n$ is isometric to $l_1^n$ (= the space of $n$-tuples $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ of reals with $\|\lambda\| = \max_{1 \leq i \leq n} |\lambda_i|$).

We recall that a sequence $\{x_i\}_{i=1}^\infty$ in a Banach space $X$ is called a monotone basis of $X$ (see [1, p. 67]) if each $x \in X$ has a unique representation $x = \sum_{i=1}^\infty \alpha_i x_i$ where $\{\alpha_i\}$ are scalars and the projections $P_n$ on $X$ defined by $P_n(\sum_{i=1}^\infty \alpha_i x_i) = \sum_{i=1}^n \alpha_i x_i$ are of norm 1.

The purpose of this note is to show that the space $c_0$ (= the space of real sequences $\rho = \{\rho_n\}_{n=1}^\infty$ which converge to 0 with $\|\rho\| = \sup_{1 \leq n} |\rho_n|$) is the minimal infinite dimensional Banach space whose dual is an $L_1$ space, namely

**Theorem 1.** Let $X$ be a separable infinite dimensional Banach space whose dual is an $L_1$ space. Then $X$ has a subspace $V$ isometric to $c_0$ such that there is a projection of norm 1 from $X$ onto $V$.

It is proved in [3, p. 67, Corollary 2] that any infinite dimensional Banach space $Y$ with $Y^* = L_1(\mu)$ has a separable infinite dimensional subspace $X$ for which $X^* = L_1(\nu)$. Hence, Theorem 1 implies the following

**Corollary 1.** Every infinite dimensional Banach space whose dual is an $L_1$ space contains a subspace isometric to $c_0$.

2. Preliminary lemmas. Let $X$ be a separable Banach space such that $X^*$ is an $L_1$ space and assume that $\{x_i\}_{i=1}^\infty$ is the monotone basis of $X$ mentioned in Proposition 1. It follows from Proposition 1 that in each subspace $E_n = \text{span } \{x_i\}_{i=1}^n$ there is a basis $\{e_i^n\}_{i=1}^n$ such that

$$\| \sum_{i=1}^n \gamma_i e_i^n \| = \max_{1 \leq i \leq n} |\gamma_i|$$

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for any real $\gamma_1, \gamma_2, \cdots, \gamma_n$. It is easy to prove (see [4, §5] and [5]) that the bases $\{e^n_i\}_{i=1}^n$, $n = 1, 2, 3, \cdots$ can be chosen such that for every $n$ and $1 \leq i \leq n$

\begin{equation}
\sum_{i=1}^{n} a^n_i e^n_i = e^{n+1}_i + a^n_i e^{n+1}_i
\end{equation}

where $\sum_{i=1}^{n} |a^n_i| \leq 1$. We define now a sequence $\{\phi_i\}_{i=1}^\infty$ of functionals on $\bigcup_{n=1}^\infty E_n$ in the following way: For each $j < n$ and $x = \sum_{i=1}^{n} b_i e^n_i$, $\phi_j(x) = b_j$. Using (2) it is easy to show that $\phi_j$ is uniquely defined and linear. Moreover, $|\phi_j(x)| = |b_j| \leq \max_{1 \leq i \leq n} |b_i| = \|x\|$ and hence, $\phi_j$ can be extended by continuity to a linear functional of norm 1 on $X$. Another property of $\phi_j$ is proved in the following

**Lemma 1.** For each $j$ $\phi_j$ is an extreme point of the unit ball of $X^*$.

**Proof.** Assume that $\phi_j = 1/(f + g)$ where $f, g \in X^*$ and $\|f\| = \|g\| = 1$. For all $n \geq j$, $f(e^n_j) = g(e^n_j)$ and if, for some $k \neq j$, $f(e^n_k) = \alpha \neq 0$ then

$f(e^n_j + (\text{sign } \alpha)e^n_k) = 1 + |\alpha| > 1$.

This contradicts the fact that $\|f\| = 1$ (since $\|e^n_j + (\text{sign } \alpha)e^n_k\| = 1$). It follows that $f(e^n_k) = 0$ for every $n \geq j$ and $k \neq j$. Obviously $g$ shares the same property and therefore $f(x) = g(x) = \phi_j(x)$ for any $x \in \bigcup_{n=1}^\infty E_n$. Since $\bigcup_{n=1}^\infty E_n$ is dense in $X$ Lemma 1 is thus proved.

**Corollary 2.** The closed linear span of $\{\phi_j\}_{j=1}^\infty$ is isometric to $l_1$.

**Proof.** $X^*$ is an $L_1(\mu)$ space and, as is well known, the extreme points of the unit ball are elements of the form $\phi = \chi_A/\mu(A)$ where $A$ is an atom and $\chi_A$ the characteristic function of $A$. It follows from Lemma 1 that $\|\sum_{i=1}^\infty \alpha_i \phi_i\| = \sum_{i=1}^\infty |\alpha_i|$ for any sequence $\{\alpha_i\}_{i=1}^\infty$ of reals. This concludes the proof.

For every $x = \sum_{j=1}^{n} b_j e^n_j \in \bigcup_{k=1}^\infty E_k$ we have $\|x\| = \max_{1 \leq j \leq n} |b_j| = \max_{1 \leq j \leq n} |\phi_j(x)| \leq \sup_{1 \leq j < \infty} |\phi_j(x)| \leq \|x\|$. It follows that for every $x \in X$

\begin{equation}
\|x\| = \sup_{1 \leq j < \infty} |\phi_j(x)|.
\end{equation}

The functionals $\phi_j$ will play an important role in the sequel.

**Lemma 2.** Let $n(k)$ and $i(k)$ be increasing sequences of positive integers with $i(k) \leq n(k)$. Then the sequence $\{e_{i(k)}^m\}_{k=1}^\infty$ forms a monotone basis in the subspace $E$ which it spans in $X$. Moreover, in each subspace $F_m = \text{span} \{e_{i(k)}^m\}_{k=1}^m$ there is a basis $\{u_i^m\}_{i=1}^m$ satisfying the following

(a) $u_i^m = e_{i(m)}^m$.

(b) For every $m$ and $1 \leq i \leq m$, $u_i^m = u_{i+1}^m + b_i u_{i+1}^m$ where $\sum_{i=1}^m |b_i| \leq 1$. 

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(c) \[ \| \sum_{i=1}^{n} \gamma_{i} u_{i}^{n} \| = \max_{1 \leq i \leq n} |\gamma_{i}| \text{ for any real } \gamma_{1}, \cdots, \gamma_{m}. \]

**Proof.** Define \( u_{1} = e_{i(1)}^{n} \) and for \( k > 1 \)

\[
\begin{align*}
  u_{k} &= e_{i(k)}^{n} & \text{if } h = k \\
  &= \sum_{j=1, j \neq i(k)}^{n(k)} b_{h,j} e_{j}^{n(k)} & \text{if } h < k \text{ and } u_{h} = \sum_{j=1}^{n(k)} b_{h,j} e_{j}^{n(k)}.
\end{align*}
\]

The properties (a) and (b) follow directly from the definition of \( u_{j}^{k} \) and from (c), while the following assertion is an easy consequence of (2): Let \( x = \sum_{i=1}^{s} a_{i} e_{i} = \sum_{i=1}^{s} b_{i} e_{i} \) where \( s > r \). Then \( a_{i} = b_{i} \) for all \( i \leq r \). (Proof by induction on \( s - r \).) In view of this fact and the definition of \( u_{j}^{k} \), we have the equality \( 1 = b_{i,j}^{(k)} \) for \( 1 \leq j < k \). Assertion (c) holds for \( m = 1 \) and, assuming its validity for \( m = k - 1 \), we get that \( \sum_{j=1}^{k-1} |b_{h,j}^{k}| \leq 1 \) for all \( 1 \leq j \leq n(k) \). (This follows from the fact that \( \{ e_{j}^{n(k)} \}_{j=1}^{n(k)} \) is the usual basis in the \( \ell^{1}(m) \text{-space } E_{n(k)} \) and that \( \| \sum_{j=1}^{k-1} \pm u_{j}^{k-1} \| = 1 \) for any choice of the signs.) In particular it follows that \( b_{p,i(j)}^{k} = 0 \) for all \( p \neq j, 1 \leq p, j \leq k \). We thus get that

\[
\max_{1 \leq j \leq k-1} |\gamma_{j}| \leq \| \sum_{j=1}^{k} \gamma_{j} u_{j}^{k} \| \leq \| \sum_{j=1}^{k-1} \gamma_{j} u_{j}^{k-1} \| = \max_{1 \leq j \leq k-1} |\gamma_{j}|.
\]

for any real \( \gamma_{1}, \cdots, \gamma_{k-1} \). The definition of \( u_{j}^{k} \) and the last inequality yield

\[
\| \sum_{j=1}^{k} \gamma_{j} u_{j}^{k} \| = \max \left\{ \| \sum_{j=1}^{k-1} \gamma_{j} u_{j}^{k} \|, |\gamma_{k}| \right\} = \max_{1 \leq j \leq k} |\gamma_{j}|.
\]

This proves (c).

Now, for any sequence \( \{ \gamma_{j} \}_{j=1}^{k+1} \) of reals let \( \sum_{j=1}^{k} \gamma_{j} e_{i(j)}^{n(j)} = \sum_{j=1}^{k} \beta_{j} u_{j}^{k} \). Then by (a), (b) and (c)

\[
\| \sum_{j=1}^{k} \gamma_{j} e_{i(j)}^{n(j)} \| = \max_{1 \leq j \leq k} |\beta_{j}| \leq \max \left\{ \max_{1 \leq j \leq k} |\beta_{j}|, |\gamma_{k+1} + \sum_{j=1}^{k} \beta_{j} b_{j}^{k}| \right\}
\]

\[
= \left\| \sum_{j=1}^{k} \beta_{j} u_{j} + \gamma_{k+1} u_{k+1} \right\| = \left\| \sum_{j=1}^{k+1} \gamma_{j} u_{j} \right\| = \left\| \sum_{j=1}^{k+1} \gamma_{j} e_{i(j)}^{n(j)} \right\|.
\]

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This inequality shows that \( \{ e_i^{(j)} \} \) is a monotone basis of \( E \) (see \([1, \text{p. 67}]\)), and Lemma 2 is proved.

By Proposition 1 and Lemma 2 any sequence \( \{ e_i^{(k)} \} \), where \( n(k) \) and \( i(k) \) are increasing and \( i(k) \leq n(k) \), span a subspace \( E \) whose dual is an \( L_1 \) space. As in the definition of the functionals \( \{ \phi_i \} \) given at the beginning of \( \S 2 \), we define functionals \( \{ \psi_j \} \) on \( E \) by \( \psi_j(\sum_{i=1}^{k} b_i u_i^k) = b_j \) if \( k \geq j \) and then extend them by continuity to \( E \). By (3) we have, for every \( x \in E, \| x \| = \sup_{1 \leq j < \infty} | \psi_j(x) | \). Since \( u_h^k = \sum_{j=1}^{n(k)} u_{h,j}^k e_j^{(k)} \) where \( b_{h,j}^k = \delta_{h,j} \) we get that \( \phi_i^{(j)}(u_h^k) = \delta_{h,j} = \psi_j(u_h^k) \) for \( 1 \leq j, h \leq k \). Hence \( \psi_j(x) = \phi_i^{(j)}(x) \) for all \( x \in E \) and thus

\[
(5) \quad \| x \| = \sup_j \| \phi_i^{(j)}(x) \|.
\]

In the sequel we will need the following well known

**Lemma 3.** Let \( \{ w_k \} \) be a sequence of elements of \( c_0 \) where \( w_k = (\omega^k, \omega^2, \ldots) \) and assume that the following conditions are satisfied:

\[
(6) \quad \omega^k = 1 \quad \text{for every } k
\]

\[
(7) \quad \sum_{k=1; k \neq i}^{\infty} | \omega_i^k | \leq 1/2.
\]

Then \( c_0 = \text{span} \{ w_k \} \).

**Proof.** Let \( U \) be the operator defined by \( U(\sum_{k=1}^{\infty} \gamma_k e_k) = \sum_{k=1}^{\infty} \gamma_k w_k \) where \( \{ e_k \} \) denotes the usual unit vector basis in \( c_0 \). Since \( U \) is given by the matrix \( [ \omega_i^k ] \), we get by (6) and (7) that \( \| I - U \| \leq 1/2 \), where \( I \) is the identity on \( c_0 \). It follows that \( U \) is invertible and hence \( c_0 = \text{span} \{ w_k \} \).

3. **Proof of Theorem 1.** Let \( X \) be a separable Banach space whose dual is an \( L_1 \) space. By Proposition 1 there is a system \( \{ e_i \}_{i=1}^{\infty} \) constructed there; by the \( \omega^* \) sequential compactness of the unit ball in \( X^* \) there is a subsequence \( \{ \phi_n^{(j)} \} \) of \( \{ \phi_n \} \) which converges \( \omega^* \) to a functional \( \phi \). By Lemma 2, the subspace \( E = \text{span} \{ e_i^{(j)} \} \) has an \( L_1 \) space as a dual and if \( \psi_j \) and \( \psi \) denote the restrictions of \( \phi_n^{(j)} \) and \( \phi \) to \( E \) respectively, the following conditions are satisfied:

\[
(8) \quad \| x \| = \sup_{1 \leq j} | \psi_j(x) |,
\]

\[
(9) \quad \lim_{j \to \infty} \psi_j(x) = \psi(x),
\]
for every $x \in E$. Construct now in $E$ the bases $\{ u_j^k \}_{j=1}^k, k = 1, 2, \ldots$ satisfying (a), (b) and (c) of Lemma 2. It follows from (c) that for any choice of the signs $\| \sum_{i=1}^k \pm u_i^k \| = 1$ and therefore, for any $k$ we have

$$\sum_{i=1}^k | \psi(u_i^k) | \leq 1.$$  

Put $p(1) = 16$ and choose $q(1), 1 \leq q(1) \leq 16$, such that $| \psi(u_q^{p(1)}) | \leq 16^{-1}$. Denote $\mu_1 = \psi(u_q^{p(1)})$ and choose

$$p(2) \geq 4 \cdot 8^3 \quad \text{if } \mu_1 = 0$$
$$\geq 32^{-1} \mu_1^{-1} \quad \text{if } \mu_1 \neq 0$$

so large that

$$| \psi_h(u_q^{p(1)}) - \mu_1 | \leq 16^{-1} \quad \text{if } \mu_1 = 0$$
$$\leq 2^{-1} \mu_1 \quad \text{if } \mu_1 \neq 0$$

for all $h \geq 2^{-1} p(2)$. By (10) we can choose $q(2), 2^{-1} p(2) \leq q(2) \leq p(2)$, such that

$$| \psi(u_q^{p(2)}) | \leq 2^{-1} \cdot 8^{-2} \quad \text{if } \mu_1 = 0$$
$$\leq 2^{-1} \cdot 8^{-1} \mu_1 \quad \text{if } \mu_1 \neq 0.$$  

Put $\mu_2 = \psi(u_q^{p(2)})$. Proceeding by induction we define two increasing sequences $p(k)$ and $q(k)$ of positive integers such that if $\mu_k$ denotes $\psi(u_q^{p(k)}) = \lim_{j \to \infty} \psi_j(u_q^{p(k)})$ then the following inequalities are satisfied:

$$| \mu_k | \leq 2^{-1} \cdot 8^{-k} \quad \text{if } \mu_{k-1} = 0$$
$$\leq 32^{-1} \mu_{k-1} \quad \text{if } \mu_{k-1} \neq 0$$

for all $h \geq 2^{-1} p(k+1)$.  

$$| \psi_h(u_q^{p(k)}) - \mu_k | \leq 2^{-1} \cdot 8^{-k} \quad \text{if } \mu_{k-1} = 0$$
$$\leq 2^{-1} \mu_{k-1} \quad \text{if } \mu_{k-1} \neq 0$$

for all $h \geq 2^{-1} p(k+1)$.

$$p(k + 1) \geq q(k + 1) \geq 2^{-1} p(k + 1) > p(k).$$

Now, either (I) there is an $N$ such that $\mu_k \neq 0$ for all $k > N$, or (II) there exists a sequence $\{ t(k) \}_{k=1}^\infty$ of positive integers such that $\mu_{t(k)} = 0$ for all $k$. Suppose that (I) holds, then we define for $k > N$

$$\alpha_k = 1 - \frac{1}{\mu_{2k-1} \cdot \mu_{2k} \cdot \psi(u_q^{p(2k-1)})}$$

and
Using (11) and (12) one can easily prove that the elements $v_k$ of $E$ satisfy the following conditions:

\begin{align}
(14) \quad & \lim_{j \to \infty} \psi_j(v_k) = 0, \\
(15) \quad & \psi_{q(2k)}(v_k) = 1, \\
& |\psi_{q(i)}(v_k)| \leq 3 \cdot 2^{1-8^{2k}} \text{ if } i > 2k \\
& \leq 7^{-1} \text{ if } i = 2k - 1 \\
& \leq 0 \text{ if } i < 2k - 1.
\end{align}

It follows from (16) that

\begin{align}
\sum_{k=1}^{\infty} |\psi_{q(2k-1)}(v_k)| & \leq 1/4 \\
\sum_{k=1}^{\infty} |\psi_{q(2n)}(v_k)| & \leq 1/4 
\end{align}

for $n = 1, 2, \ldots$.

We apply now (4) and Lemma 2 to the space $E = \text{span}\{u_k^\alpha\}_{k=1}^\infty$ where $\{u_k^\alpha\}_{k=1}^\infty$, $\psi_k$, $p(k)$, and $q(k)$ play the role of $\{c_i^\alpha\}_{i=1}^\infty$, $\phi_k$, $n(k)$, and $i(k)$ of Lemma 2, respectively, and we get that for each $u \in U = \text{span}\{u_k^\alpha\}_{k=1}^\infty$, $||u|| = \sup_{1 \leq k \leq n} |\psi_q(k)(u)|$. We claim that if $v \in V_0 = \text{span}\{v_k\}_{k=1}^\infty$ then $||v|| = \sup_{1 \leq k \leq n} |\psi_{q(2k)}(v)|$. Indeed, if $v = \sum_{i=1}^n \lambda_i v_i$ and $|\lambda_s| = \max_{1 \leq a \leq n} |\lambda_s|$ for some $1 \leq s \leq n$ then by (17), for any $k$, $|\psi_{q(2k-1)}(v)| \leq 1/4 |\lambda_s|$ while $|\psi_{q(2k)}(v)| \geq 3/4 |\lambda_s|$, and this proves the assertion. Define now a sequence $\{w_k\}_{k=1}^\infty$ in $c_0$ by $w_k = (\psi_q(2k)(v_k), \psi_q(4)(v_k), \psi_q(8)(v_k), \ldots)$. Our discussion above shows that the subspace $V_0 = \text{span}\{v_k\}_{k=1}^\infty$ of $X$ is isometric to the subspace $W = \text{span}\{w_k\}_{k=1}^\infty$ of $c_0$ (the transformation $T: V_0 \to W$ defined by $T(\sum_{i=1}^n \gamma_i v_i) = \sum_{i=1}^n \gamma_i w_i$ for any $\sum_{i=1}^n \gamma_i v_i \in V_0$ can be extended to the desired isometry). The sequence $\{w_k\}_{k=1}^\infty$ satisfies, by (15) and (17), the assumptions of Lemma 3 and hence $W = c_0$. It is easy to see that (15), (16), and (17) yield the existence of a sequence $\{y_k\}_{k=1}^\infty$ in $V_0$ such that $\psi_{q(2j)}(y_k) = \delta_{j,k}$, $k, j = 1, 2, \ldots$. Consider the sequence $\{y_{2k-1} - y_{2k}\}_{k=1}^\infty$; obviously it spans in $V_0$ a subspace $V$ isometric to $c_0$ and

\begin{align}
(18) \quad & \left\| \sum_{k=1}^{\infty} \lambda_k (y_{2k-1} - y_{2k}) \right\| = \max_{1 \leq k} |\lambda_k|
\end{align}

for any sequence $\{\lambda_k\}$ in $c_0$. We note that $\psi_j$ is defined to be the restriction of $\phi_n(j)$ to $E$ and that $\{n(j)\}$ are chosen such that $\lim_{j \to \infty} \phi_n(j)(x)$ exists for all $x \in X$. Put $\phi_j = \phi_n(q(2j))$ and define for every $x \in X$.
\[ Px = 2^{-1} \sum_{j=1}^{\infty} (\phi_{2j-1}(x) - \phi_{2j}(x)) (y_{2j-1} - y_{2j}). \]

The transformation \( P \) is obviously a projection from \( X \) onto \( V \) since \( \lim_{j \to \infty} (\phi_{2j-1}(x) - \phi_{2j}(x)) = 0 \) for all \( x \in X \). Moreover, \( \|Px\| = 2^{-1} \sup_{x \in D} |\phi_{2j-1}(x) - \phi_{2j}(x)| \leq \|x\| \) by Corollary 2, and hence \( \|P\| = 1 \).

It remains to consider the possibility (II). If (II) holds, we define \( v_k = \psi_{q(k)}(v_k) \) and \( w_k = (\psi_{q(k)}(v_k), \psi_{q(k+1)}(v_k), \ldots) \) in \( c_0 \). Again, let \( W = \text{span} \{ w_k \}_{k=1}^{\infty} \) and \( V_0 = \text{span} \{ v_k \}_{k=1}^{\infty} \), it is easy to show that \( V_0 \) and \( W \) are isometric and that the sequence \( \{w_k\} \) satisfies the assumptions of Lemma 3, and hence \( V_0 \) is isometric to \( c_0 \). The construction of \( V \) and \( P \) is done exactly in the same way. This proves Theorem 1.

4. Concluding remarks. A. Let \( X \) be a separable Banach space whose dual is an \( L_1 \) space and assume that the unit cell of \( X \) has at least one extreme point. Then \( X \) contains a subspace \( V \) isometric to \( c_0 \) (the space of convergent sequences \( X = \{x^n\} \) of reals with \( \|x^n\| = \sup_n |x^n| \)) such that there is a projection of norm 1 from \( X \) onto \( V \). This assertion follows from the following considerations (which are only a sketch of the proof): It was observed by Semadeni [6] that a space satisfying the assumptions of our assertion is isometric to a space \( \alpha(s) \) of affine continuous functions on a Choquet simplex \( s \). Using the methods of [5] one can show that if \( e \) denotes the unit function on \( s \) then the bases \( \{e^n_i\}_{i=1}^{n} \) \( n = 1, 2, \ldots \) can be chosen such that \( e = e_1^1 \) and \( \phi_j(e) = 1 \) for all \( j \). The proof of Theorem 1 now shows that the subspace \( V \) spanned by \( \{y_k\}_{k=1}^{\infty} \) and \( e_1^1 \) in \( X \) is isometric to \( c_0 \). The desired projection is given by \( P(x) = \phi(x) e_1^1 + \sum_{j=1}^{\infty} (\phi_j(x) - \phi(x)) y_j. \)

B. Given a sequence \( \{m(i)\}_{i=1}^{\infty} \) of positive integers, the space \( X \) and the system \( \{e^n_i\}_{i=1}^{n} \) of bases described at the beginning of §2 we define \( x_i = e_i^{s(k)} \) if \( s(k-1) < i \leq s(k) \), where \( s(0) = 0 \) and \( s(k) = \sum_{i=1}^{k} m(i) \). The sequence \( \{x_i\}_{i=1}^{\infty} \) satisfies the property that

\[
\sum_{i=s(k)+1}^{s(k+1)} \lambda_i x_i \leq \max_{s(k) < i \leq s(k+1)} |\lambda_i|
\]

and by the proof of Lemma 2 it forms a basis in \( X \). Call the bases \( \{x_i\} \) and \( \{y_i\} \) equivalent if the convergence of \( \sum_{i=1}^{n} \alpha_i x_i \) is equivalent to the convergence of \( \sum_{i=1}^{n} \alpha_i y_i \) for any real sequence \( \{\alpha_i\}_{i=1}^{\infty} \). It follows that if the basis \( \{e_i^n\}_{i=1}^{\infty} \) of \( X \) is not equivalent to the unit vector basis of \( c_0 \) then by choosing the sequence \( \{m(i)\} \) suitably one can construct infinitely many mutually nonequivalent bases of the form \( \{x_i\}_{i=1}^{\infty} \).
References


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