# THE TANGENT BUNDLE OF THE LONG LINE

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We prove the following surprise.

THEOREM. *The tangent bundle of the long line is not trivial.*

PROOF. The long line $L$ is defined in Hocking and Young [1], for example. It is easy to check that $L$, with the topology defined there, supports the structure of a $C^\infty$ manifold. In fact, it is proved in [2] that $L$ (there called the Alexandroff Half Line) can be made into a real analytic manifold (I thank the referee for this reference). Let us recall what it means for a real vector bundle $E$ to be trivial. The bundle $E \xrightarrow{\pi} X$ is trivial if there is a bundle map $\phi: E \to \xi^n$ covering the identity: $X \to X$, where $\xi^n$ is the trivial $\boldsymbol{R}^n$ bundle over $X$ and $n$ is the dimension of the fibre of $E$. But this is easily seen to be equivalent to the existence of a bundle map $\tilde{\phi}$: $E \to \boldsymbol{R}^n$ where $\boldsymbol{R}^n$ is considered as a bundle over a point $*$. (If $\omega$: $\xi^n \to \boldsymbol{R}^n$ is projection onto the coordinate of each fibre, then $\tilde{\phi} = \omega \circ \phi$ is such a map. If $\tilde{\phi}$ is given then $\tilde{\phi}^{-1}\boldsymbol{R}^n = \xi^n$ and $E \cong \tilde{\phi}^{-1}\boldsymbol{R}^n$ where $\tilde{\phi}^{-1}$ means the pull back under $\tilde{\phi}$.) So let $TL$ be the tangent bundle of $L$ and $\phi$ be a $C^\infty$ trivialization

$$
\begin{array}{ccc}
TL & \xrightarrow{\phi} & \boldsymbol{R}^1 \\
\pi \downarrow & & \downarrow \\
L & \longrightarrow & *.
\end{array}
$$

Then $g(x, y) = \phi(x) \cdot \phi(y)$ defines a $C^\infty$ positive definite inner product on the fibres of $TL$ ($\phi$ is an isomorphism on each fibre). Thus $L$ would be a Riemannian manifold. But it is well known (see Kobayashi-Nomizu [3, p. 166]) that a Riemannian manifold (not necessarily assumed paracompact) has a metric space structure which defines the manifold topology. Now it is well known that $L$ is not paracompact, and hence is not a metric space. This contradiction proves the theorem.

## REFERENCES


1. J. G. Hocking and G. S. Young, *Topology*, Addison-Wesley, Reading, Mass., 1961.

2. H. Kneser, *Analytische Struktur und Abzählbarkeit*, Ann. Acad. Sci. Fenn. Ser. A.I. No. 251/5 (1968), 8 pp.

3. S. Kobayashi and K. Nomizu, *Foundations of differential geometry*. Vol. I, Interscience, New York, 1963.


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