THE INTERSECTION OF INDECOMPOSABLE CONTINUA

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The theme of this paper is to reveal a fundamental result concerning the 2-finished sum of compact continua, one of which is hereditarily indecomposable.

**Definition.** The set $M$ is the $k$-finished sum of a set of subcontinua, $\{M_1, \ldots, M_k\}$, if and only if

$$M = \bigcup_{i=1}^{k} M_i$$

and for each fixed $j$, $1 \leq j \leq k$,

$$M_j - \bigcup_{1 \leq i \leq k; i \neq j} M_i \neq \emptyset.$$

In this paper we shall consider the space $S$ to be a Moore space satisfying Axiom 0 and Axiom 1 of R. L. Moore. The boundary of a set $D$ with respect to a set $M$ will be denoted by $F(D)_M$.

**Theorem.** If $M$ is the 2-finished sum of compact continua, $M_1$ and $M_2$, such that $M_1$ is hereditarily indecomposable and $M_1 \cap M_2 \neq \emptyset$, then there exists at least one point in $M_1 \cap M_2$ which is a limit point of both $(M_1 - M_2)$ and $(M_2 - M_1)$.

**Proof.** Since $M_1 \cap M_2 \neq \emptyset$, then $M$ is a compact continuum. Suppose that no point of $M_1 \cap M_2$ is a limit point of both $H = M_1 - M_2$ and $K = M_2 - M_1$. That is, suppose $F(H)_M \cap F(K)_M = \emptyset$. The supposition implies that $\overline{H} \cap \overline{K} = \emptyset$.

Let $T$ be a component of $H$. The reference [1] implies that there exists a point $p \in F(H)_M \cap T$. Also it is noted that $T \subset M_1$ since $M_1$ is a closed point set containing $T$. Since $H$ is a domain relative to $M$ then $p \in H$ and $p \in M_2$.

The point $p \in \overline{K}$ since $p \in M_1 \cap F(H)_M$. Thus the point set $(M_2 - \overline{K}) = M_2 \cap (M - \overline{K})$ is a domain relative to $M_2$ containing $p$. Let $L$ be the component of $(M_2 - \overline{K})$ containing the point $p$. The reference [1] implies that there exists a point $q \in F(K)_M \cap \overline{L}$. The point $q \in T$ for if so then $q \in F(H)_M$ which contradicts the supposition. The point set $\overline{T} \subset M_1$ since $M_1$ is a closed point set containing $L$.

By the definition of both $\overline{T}$ and $\overline{L}$ we know that each is a subcontinuum of $M_1$. Then since $p \in \overline{T} \cap \overline{L}$ the point set $\overline{T} \cap \overline{L}$ is a sub-
continuum of $M_1$. Since $q \in \overline{T}$ then $\overline{L} - \overline{T} \neq \emptyset$. Also since $T \subset H$ and $\overline{L} \cap H = \emptyset$, then $\overline{T} - \overline{L} \neq \emptyset$. Thus the point set $\overline{T} \cup \overline{L}$ is a decomposable subcontinuum of $M_1$. This contradicts $M_1$ being hereditarily indecomposable, and thus the theorem is proved.

**Bibliography**


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