SHORTER NOTES

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WEAKLY CLOSED MAXIMAL TRIANGULAR ALGEBRAS ARE HYPERREDUCIBLE

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In this note we observe that the assertion in the title follows easily from known results.

Recall that an algebra $\mathfrak{R}$ of bounded linear operators on a Hilbert space is called [1] *triangular* if $\mathfrak{R}\cap\mathfrak{R}^*$ is a maximal abelian von Neumann algebra. A triangular algebra is *maximal* if it is not properly contained in another triangular algebra; it is *hyperreducible* if the projections onto its invariant subspaces generate a maximal abelian von Neumann algebra.

Now the theorem stated in the title can be proven as follows. Let $\mathfrak{R}$ be a weakly closed maximal triangular algebra. Since $\mathfrak{R}$ is maximal the lattice of invariant subspaces of $\mathfrak{R}$ is totally ordered [**1**, Lemma 2.3.3]. It follows from Theorem 2 of [2] that $\mathfrak{R}$ is *reflexive*, i.e., $\mathfrak{R}$ contains every operator which leaves invariant every invariant subspace of $\mathfrak{R}$. Hence, by Lemma 5.1 of [**3**], $\mathfrak{R}$ is hyperreducible.

This result shows one reason that hyperreducible maximal triangular algebras are "nice" (see [**1**]).

Note that the proof of Lemma 5.1 of [**3**] shows that every reflexive triangular algebra (not just those with totally ordered invariant subspace lattices), is hyperreducible. Thus it follows (by a proof similar to the above) from Theorem 3 of [2] that a weakly closed triangular algebra whose lattice of invariant subspaces is the direct product of a family of totally ordered lattices is hyperreducible.

In view of these results it seems reasonable to conjecture that every weakly closed triangular algebra is hyperreducible. A proof of this conjecture would be a partial answer to question (i) of [2].

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