A NOTE ON INTERPOLATION

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The purpose of this note is to give a direct proof of the following theorem: [2].

Let A be a uniform algebra on a compact Hausdorff space X and E a closed subset of X. If $\operatorname{Re} A \mid E = C_R(E)$, *then* $A \mid E = C(E)$.

PROOF. By a result of Glicksberg [1] we have $A \mid E = C(E)$ if and only if there is a constant c such that $\|\mu_E\| \leqq c\|\mu_{E'}\|$ whenever μ is a complex Borel measure on X orthogonal to A and μ_E, $\mu_{E'}$ are the restriction of μ to E and E' (the complement of E) respectively. Now let μ be a measure orthogonal to A. Choose $f \in C_R(E)$, $0 \leqq f \leqq 2\pi$, such that $\int_E e^{if} d\mu$ is ϵ-close to $\|\mu_E\|$. By applying the open mapping theorem to $A \to \operatorname{Re} A \mid E = C_R(E)$ one sees that there exists a constant c_1 such that for every $g \in C_R(E)$ there is a $\phi \in A$, $\operatorname{Re} \phi \mid E = g$, and $\|\phi\| \leqq c_1 \|g\|$. In particular if $\operatorname{Re} \phi \mid E = f$ (as above) we have that

$$\int_E e^{i\phi} d\mu \text{ is close to } c_2\|\mu_E\| \quad \text{where} \quad \exp(-2\pi c_1) \leqq c_2 \leqq \exp(2\pi c_1).$$

Indeed if $\phi = \tilde{f} + ig$, $\tilde{f}$ a real extension of f to X then $\left|\int_E e^{if-g} d\mu\right| \geqq [\|\mu_E\| - \epsilon]\exp(-2\pi c_1)$. On the other hand since $e^{i\phi} \in A$ we have that

$$\int_E e^{i\phi} d\mu = -\int_{E'} e^{i\phi} d\mu, \quad \text{and} \quad \left|\int_{E'} e^{i\phi} d\mu\right| \leqq \exp(2\pi c_1)\|\mu_{E'}\|.$$

Combining the above results we get

$$\|\mu_E\| - \epsilon \leqq \exp(4\pi c_1)\|\mu_{E'}\| \text{ as required.}$$

REFERENCES

1. I. Glicksberg, *Measures orthogonal to algebras and sets of antisymmetry*, Trans. Amer. Math. Soc. **105** (1962), 415–435. MR **30** #4164.

2. S. J. Sidney and E. L. Stout, *A note on interpolation*, Proc. Amer. Math. Soc. **19** (1968), 380–382. MR **36** #6944.

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