A NOTE ON THE CARDINALITY OF THE MEDVEDEV LATTICE

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In [1] Rogers discusses the Medvedev lattice of mass problems and states that its cardinality is unknown. In this note we simply show

THEOREM. *The Medvedev lattice has* 2^c *members; in fact there is a set of pairwise incomparable elements of cardinality* 2^c.

PROOF. Let $\mathcal{A} \subseteq N^N$ be a set of cardinality c of functions of incomparable Turing degree [2]. Let A be a family of subsets of $\mathcal{A}$ of cardinality 2^c which are incomparable with respect to inclusion (such a family exists by identifying $\mathcal{A}$ with the reals and letting A be the family of all Hamel bases—this observation is due to Nerode). Then distinct members of A have incomparable M-degree for suppose $\mathcal{B}_1$ and $\mathcal{B}_2$ are in A and are distinct and further suppose that there is a recursive operator Φ with $\Phi(\mathcal{B}_2) \subseteq \mathcal{B}_1$. Let $f \in \mathcal{B}_2 - \mathcal{B}_1$ (since $\mathcal{B}_2$ is not a subset of $\mathcal{B}_1$) then $\Phi(f) \neq f$ and both are in $\mathcal{A}$ contradicting the fact that the elements of $\mathcal{A}$ have incomparable Turing degree.

This result was also found independently by Elizabeth Jockusch and John Stillwell.

REFERENCES

1. H. Rogers, Jr., "Some problems of definability in recursive function theory" in *Sets, models and recursion theory*, Proc. Summer School Math. Logic and Tenth Logic Colloq. (Leicester, 1965), North-Holland, Amsterdam, 1967, pp. 183–201. MR **36** #6286.

2. G. E. Sacks, *Degrees of unsolvability*, Ann. of Math. Studies, no. 55, Princeton Univ. Press, Princeton, N. J., 1963. MR **32** #4013.

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