A NOTE ON THE CARDINALITY OF THE MEDVEDEV LATTICE

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In [1] Rogers discusses the Medvedev lattice of mass problems and states that its cardinality is unknown. In this note we simply show

Theorem. The Medvedev lattice has $2^c$ members; in fact there is a set of pairwise incomparable elements of cardinality $2^c$.

Proof. Let $\mathcal{A} \subseteq \mathcal{N}^N$ be a set of cardinality $c$ of functions of incomparable Turing degree [2]. Let $A$ be a family of subsets of $\mathcal{A}$ of cardinality $2^c$ which are incomparable with respect to inclusion (such a family exists by identifying $\mathcal{A}$ with the reals and letting $A$ be the family of all Hamel bases—this observation is due to Nerode). Then distinct members of $A$ have incomparable $M$-degree for suppose $\mathcal{B}_1$ and $\mathcal{B}_2$ are in $A$ and are distinct and further suppose that there is a recursive operator $\Phi$ with $\Phi(\mathcal{B}_2) \subseteq \mathcal{B}_1$. Let $f \in \mathcal{B}_2 - \mathcal{B}_1$ (since $\mathcal{B}_2$ is not a subset of $\mathcal{B}_1$) then $\Phi(f) \neq f$ and both are in $\mathcal{A}$ contradicting the fact that the elements of $\mathcal{A}$ have incomparable Turing degree.

This result was also found independently by Elizabeth Jockusch and John Stillwell.

References


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