AN ELEMENTARY PROOF OF THE TRIOD THEOREM

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Professor R. L. Moore proved in [1] that there exists only a countable number of mutually exclusive simple triods in the plane. The following simpler proof should be of interest.

DEFINITION 1. The plane continuum T is a simple triod if it is the sum of three arcs OX, OY, and OZ such that no two of them have any point in common except 0.

THEOREM 1. *There exists only a countable number of mutually exclusive simple triods in the plane.*

PROOF. Let $\mathcal{R}=\{R_i\}_{i=1}^{\infty}$ be a countable collection of open discs in the plane which forms a base for the usual topology. Let T be a simple triod in the plane and let $R(T, 1)$ be an element of $\mathcal{R}$ which contains 0 such that $R(T, 1)\cap\{X, Y, Z\}=\emptyset$. Let X_T, Y_T, and Z_T be the first points of the sets $(OX)\cap \mathrm{Bd}(R(T, 1))$, $(OY)\cap \mathrm{Bd}(R(T, 1))$ and $(OZ)\cap \mathrm{Bd}(R(T, 1))$, respectively. The continuum $(OX_T)\cup(OY_T)\cup(OZ_T)$ is a simple triod which is a subset of T. It follows from the Jordan Curve Theorem that $R(T, 1)-[(OX_T)\cup(OY_T)\cup(OZ_T)]=I(T, 2)\cup I(T, 3)\cup I(T, 4)$, where $\{I(T, n)\}_{n=2}^{4}$ are pairwise disjoint open sets. Let $\{R(T, n)\}_{n=2}^{4}$ be elements of $\mathcal{R}$ contained in $\{I(T, n)\}_{n=2}^{4}$, respectively. The triod T is thus associated with a quadruple of elements of $\mathcal{R}$.

Let T' be a simple triod such that $T'\cap T=\emptyset$ and let $\{R(T', n)\}_{n=1}^{4}$ be the quadruple of elements of $\mathcal{R}$ associated with T'. Since $T'\cap T=\emptyset$, it follows that if $R(T, 1)=R(T', 1)$ then two of the sets $\{I(T', n)\}_{n=2}^{4}$ must be a subset of one of the sets $\{I(T, n)\}_{n=2}^{4}$. Because of the method used to select the quadruple associated with a given triod, it follows that the two quadruples $\{R(T, n)\}_{n=1}^{4}$ are not identical. It follows that any collection of mutually exclusive simple triods can be placed in 1-1 correspondence with a subset of the collection of all quadruples of elements of $\mathcal{R}$ and hence is a countable collection.

REFERENCE

1. R. L. Moore, *Foundations of point set theory*, Amer. Math. Soc. Colloq. Publ., vol. 13, Amer. Math Soc., Providence, R.I., 1932; rev. ed., 1962. MR 27 #709.

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